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APPLICATION OF PARAMETER PLANE TECHNIQUES TO CONTROL SYSTEM COMPENSATION (IDENTICAL DOUBLE SECTION CASCADE COMPENSATION)

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APPLICATION OF PARAMETER PLANE TECHNIQUES TO CONTROL SYSTEM COMPENSATION

(IDENTICAL DOUBLE SECTION CASCADE COMPENSATION)

by

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[redacted]

Submitted in partial fulfilment
for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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May 1966

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ABSTRACT

Parameter plane techniques are applied to compensation of third order control systems by extending the principle to include the quadratic form of the parameters.

Equations for constant bandwidth curves on the parameter plane are developed.

Recursive equations for the extension of these methods to higher ordered systems are presented.

A computer program for the computation and plotting of constant zeta, omega and bandwidth loci is written.

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1. Introduction

Until 1959 when Mitrovic [1] introduced what is now called the Coefficient Plane, analysis and compensation of control systems were accomplished primarily by frequency response and root locus techniques. With the advent of the Coefficient Plane there was then a method available which allowed the control engineer to determine the root locations of a plant as two coefficients of the characteristic equation were varied.

At about the same time E.R. Ross and T.C. Warren [2] in conjunction with Dr. G.J. Thaler published a numerical method of determining single and multiple section cascade compensation while maintaining a constant error coefficient. Shortly thereafter in 1960 C.D. Pollak [3] and Dr. Thaler published yet a different numerical method for stating compensator pole locations for a given gain.

In 1964, Siljak [4] extended Mitrovic's work to apply to problems in which the system variables or parameters appeared as a product in one or more coefficients. This paper will further extend the Parameter Plane concept to include the quadratic form of the coefficients. Specifically the case of:

$$\sum_{k=0}^{n} a_k s^k = 0 \tag{1-1}$$

where:

$$a_k = b_k \alpha^2 + c_k \alpha + d_k \alpha\beta + e_k \beta + f_k \beta^2 + g_k \tag{1-2}$$

and:

$b_k, c_k, d_k, e_k, f_k, g_k$ are real and constant

α and β are real and variable

These curves which will be plotted on the $\alpha - \beta$ or Parameter Plane will specify two of the roots of Equation (1-1) for any choice of α and β.

2. Concepts of the Parameter Plane

The Parameter Plane was derived by a study of the characteristic equations of control systems where two parameters were allowed to vary. Here a presentation of the basic ideas of Siljak [4] will be studied.

Consider a characteristic equation of the form:

$$\sum_{k=0}^{n} a_k s^k = 0 \tag{2-1}$$

where the coefficients $a_1, a_2, \ldots, a_n$ are real and s is the Laplace Transform variable:

$$s = \sigma + j\omega = -\zeta\omega + j\omega\sqrt{1-\zeta^2} \tag{2-2}$$

By forming powers of s it is shown [4] that:

$$s^k = \omega^k T_k(-\zeta) + j\sqrt{1-\zeta^2}\; U_k(-\zeta) \tag{2-3}$$

where:

$$\begin{aligned} T_k(-\zeta) &= (-1)^k T_k(\zeta) \\ U_k(-\zeta) &= (-1)^{k+1} U_k(\zeta) \end{aligned} \tag{2-4}$$

The U_k and T_k are the first and second kind of Chebyshev functions and are generated by the relations [4] :

$$\begin{aligned} T_{k+1}(\zeta) - 2\zeta T_k(\zeta) + T_{k-1}(\zeta) &= 0 \\ U_{k+1}(\zeta) - 2\zeta U_k(\zeta) + U_{k-1}(\zeta) &= 0 \end{aligned} \tag{2-5}$$

with:

$$T_0(\zeta) = 1 \qquad U_0(\zeta) = 0$$

$$T_1(\zeta) = 0 \qquad U_1(\zeta) = -1$$

Taking Equation (2-1), substituting Equation (2-3) in it, then equating the real and imaginary parts to zero separately and employing Equations (2-4) and (2-5) we will obtain the expressions in terms of Chebyshev functions:

$$\sum_{k=0}^{n} (-1)^k a_k \omega^k U_{k-1}(\xi) = 0 \qquad (2\text{-}6)$$

$$\sum_{k=0}^{n} (-1)^k a_k \omega^k U_k(\xi) = 0 \qquad (2\text{-}7)$$

Now, assuming as Siljak did, that the a_k coefficients of Equation (2-1) are linear functions of α and β :

$$a_k = b_k \alpha + c_k \beta + d_k \qquad (2\text{-}8)$$

with:

b_k, c_k, d_k constant and real

α and β variable and real

we may then substitute Equation (2-8) into Equations (2-6) and (2-7) and obtain:

$$\begin{aligned} B_1 \alpha + C_1 \beta &= -D_1 \\ B_2 \alpha + C_2 \beta &= -D_2 \end{aligned} \qquad (2\text{-}9)$$

where:

$$\begin{aligned} B_1 &= \sum_{k=0}^{n} (-1)^k b_k \omega^k U_{k-1} & B_2 &= \sum_{k=0}^{n} (-1)^k b_k \omega^k U_k \\ C_1 &= \sum_{k=0}^{n} (-1)^k c_k \omega^k U_{k-1} & C_2 &= \sum_{k=0}^{n} (-1)^k c_k \omega^k U_k \\ D_1 &= \sum_{k=0}^{n} (-1)^k d_k \omega^k U_{k-1} & D_2 &= \sum_{k=0}^{n} (-1)^k d_k \omega^k U_k \end{aligned} \qquad (2\text{-}10)$$

Now Equations (2-9) may be solved simultaneously for α and β giving:

$$\alpha = \frac{C_1D_2 - C_2D_1}{B_1C_2 - B_2C_1} \qquad \beta = \frac{B_2D_1 - B_1D_2}{D_1C_2 - B_2C_1} \tag{2-11}$$

Equations (2-11) now give α and β in terms of ζ and ω and either may be fixed and the other varied from zero to infinity which will give a curve on the $\alpha - \beta$ plane. This may be repeated for as many values as needed giving an entire family of curves on the $\alpha - \beta$ plane which completely describe the behavior of a pair of complex roots. These curves will usually be constant zeta or constant omega loci.

If $s = \sigma$ is substituted in Equation (2-1) and the same procedure is followed, curves of constant sigma or values of s along the real axis may be obtained.

This then is the general procedure to be followed in the development which follows for the quadratic case.

3. Cascade Compensation of Third Order Systems

The initial investigation of the $\alpha - \beta$ quadratic case is made by attempting to compensate a specific third order system with two identical sections of cascade compensators maintaining a constant velocity coefficient. The uncompensated plant, Figure 3-1, is chosen so as to afford not only lead, but lag compensation, and with this selection, the system gain is set to place the complex roots at the stability limit, which also insures the need for double section compensation over a large range of zeta and omega. Establishing the generalized compensated plant, Figure 3-2, the characteristic equation is determined:

$$z^2s^5 + (6z^2 + 2pz^2)s^4 + (5z^2 + 12pz^2)s^3 + (10pz^2 + 6p^2z^2 + 20p^2)s^2 + (5p^2z^2 + 60p^2z)s + 30p^2z^2 = 0 \qquad (3\text{-}1)$$

Simplifying this equation and making a substitution of variables:

$$\beta = p \qquad \alpha = p/z \qquad (3\text{-}2)$$

an equation in terms of α and β is obtained:

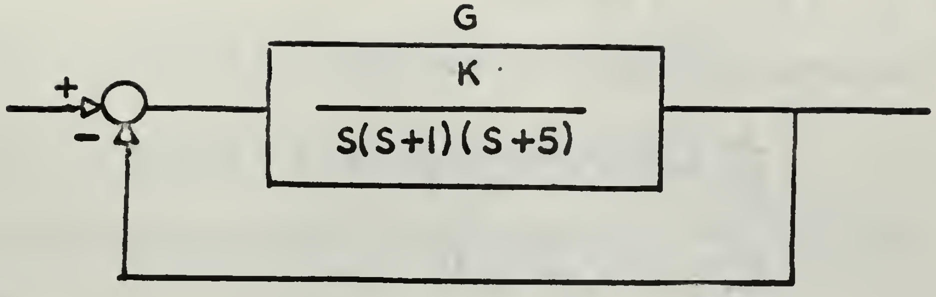
$$s^5 + (6 + 2\beta)s^4 + (5 + 12\beta + \beta^2)s^3 + (10\beta + 6\beta^2 + 30\alpha^2)s^2 + (5\beta^2 + 30\alpha\beta)s + 30\beta^2 = 0 \qquad (3\text{-}3)$$

and when the terms are arranged in terms of the parameters:

$$\alpha^2(30s^2) + \alpha\beta(60s) + \beta(2s^4 + 12s^3 + 10s^2) + \beta^2(s^3 + 6s^2 + 5s + 30) + (s^5 + 6s^4 + 5s^3) = 0 \qquad (3\text{-}4)$$

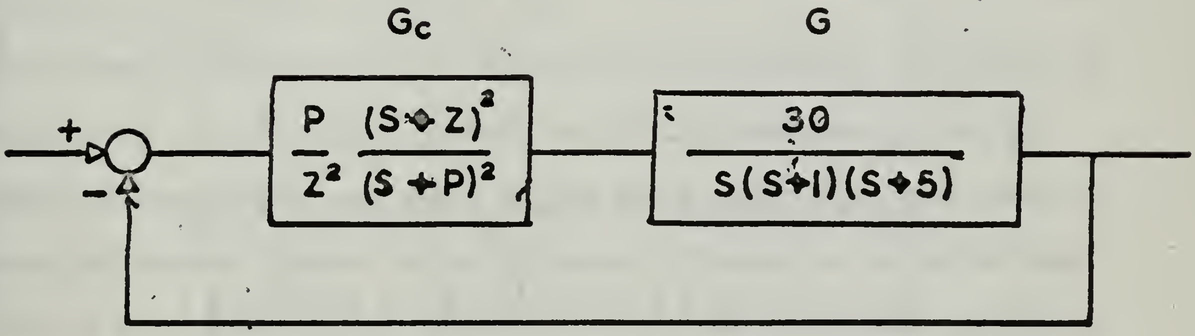
which shows the general coefficient form:

$$\sum_{k=0}^{n} (b_k\alpha^2 + c_k\alpha + d_k\alpha\beta + e_k\beta + f_k\beta^2 + g_k) = 0 \qquad (3\text{-}5)$$



UNCOMPENSATED SYSTEM

FIG. 3-1



COMPENSATED SYSTEM

FIG. 3-2

or:

$$\sum_{k=0}^{n} a_k s^k = 0 \qquad (3\text{-}6)$$

where all of the c_k's are zero.

Employing Equations (2-6) and (2-7)

$$\sum_{k=0}^{n} (-1)^k a_k \omega^k U_{k-1}(\xi) = 0 \qquad (2\text{-}6)$$

$$\sum_{k=0}^{n} (-1)^k a_k \omega^k U_k(\xi) = 0 \qquad (2\text{-}7)$$

Equation (3-4) may be split into a real and imaginary part in terms of ξ and ω :

$$\alpha^2 B_{21} + \alpha\beta D_1 + \beta E_{11} + \beta^2 F_{21} + G_1 = 0 \qquad (3\text{-}7)^1$$

$$\alpha^2 B_{22} + \alpha\beta D_2 + \beta E_{12} + \beta^2 F_{22} + G_2 = 0 \qquad (3\text{-}8)$$

where:

$$B_{21} = 30\omega^2 U_1(\xi) \qquad (3\text{-}9)$$

$$B_{22} = 30\omega^2 U_2(\xi) \qquad (3\text{-}10)$$

$$D_1 = -60\omega U_o(\xi) \qquad (3\text{-}11)$$

$$D_2 = -60\omega U_1(\xi) \qquad (3\text{-}12)$$

$$E_{11} = 2\omega^4 U_3(\xi) - 12\omega^3 U_2(\xi) + 10\omega^2 U_1(\xi) \qquad (3\text{-}13)$$

$$E_{12} = 2\omega^4 U_4(\xi) - 12\omega^3 U_3(\xi) + 10\omega^2 U_2(\xi) \qquad (3\text{-}14)$$

---

[1] The second number on the double subscripted variables and the number of the single subscripted variables indicates the real(1) or imaginary(2) part of the equation.

$$F_{21} = -\omega^3U_2(\xi) + 6\omega^2U_1(\xi) - 5\omega U_o(\xi) + 30U_{-1}(\xi) \qquad (3\text{-}15)$$

$$F_{22} = -\omega^3U_3(\xi) + 6\omega^2U_2(\xi) - 5\omega U_1(\xi) + 30U_o(\xi) \qquad (3\text{-}16)$$

$$G_1 = -\omega^5U_4(\xi) + 6\omega^4U_3(\xi) - 5\omega^3U_2(\xi) \qquad (3\text{-}17)$$

$$G_2 = -\omega^5U_5(\xi) + 6\omega^4U_4(\xi) - 5\omega^3U_3(\xi) \qquad (3\text{-}18)$$

Further collecting the terms of Equations (3-7) and (3-8) we obtain:

$$\alpha^2B_{21} + \alpha P_1 + Q_1 = 0 \qquad (3\text{-}19)$$

$$\alpha^2B_{22} + \alpha P_2 + Q_2 = 0 \qquad (3\text{-}20)$$

where:

$$P_1 = \beta D_1 \qquad (3\text{-}21)$$

$$P_2 = \beta D_2 \qquad (3\text{-}22)$$

$$Q_1 = \beta E_{11} + \beta^2F_{21} + G_1 \qquad (3\text{-}23)$$

$$Q_2 = \beta E_{12} + \beta^2F_{22} + G_2 \qquad (3\text{-}24)$$

We now have obtained a set of two non-linear algebraic equations in terms of ξ and ω which must be solved simultaneously for the correct values of α and β. To do this, the method with the best chance of sucess appears to be Sylvester's Method in which we form a set of four equations by taking the original Equations (3-19) and (3-20) and forming two more by a multiplication with α giving:

$$\alpha^2B_{21} + \alpha P_1 + Q_1 = 0 \qquad (3\text{-}19)$$

$$\alpha^2B_{22} + \alpha P_2 + Q_2 = 0 \qquad (3\text{-}20)$$

$$\alpha^3 B_{21} + \alpha^2 P_1 + \alpha Q_1 = 0 \tag{3-25}$$

$$\alpha^3 B_{22} + \alpha^2 P_2 + \alpha Q_2 = 0 \tag{3-26}$$

Now placing these equations in matrix form:

$$\begin{bmatrix} 0 & B_{21} & P_1 & Q_1 \\ 0 & B_{22} & P_2 & Q_2 \\ B_{21} & P_1 & Q_1 & 0 \\ B_{22} & P_2 & Q_2 & 0 \end{bmatrix} \begin{bmatrix} \alpha^3 \\ \alpha^2 \\ \alpha \\ 1 \end{bmatrix} = 0 \tag{3-27}$$

we may say that if the α 's are not zero then:

$$\begin{vmatrix} 0 & B_{21} & P_1 & Q_1 \\ 0 & B_{22} & P_2 & Q_2 \\ B_{21} & P_1 & Q_1 & 0 \\ B_{22} & P_2 & Q_2 & 0 \end{vmatrix} = 0 \tag{3-28}$$

Expanding this determinant we obtain:

$$\begin{aligned} -B_{21}^2Q_2^2 + B_{21}B_{22}Q_1Q_2 + P_1P_2B_{21}Q_2 - P_1^2Q_2B_{22} + Q_1Q_2B_{21}B_{22} \\ -Q_1^2B_{22}^2 - Q_1P_2^2B_{21} + Q_1P_1P_2B_{22} = 0 \end{aligned} \tag{3-29}$$

Substituting Equations (3-21) through (3-24) in Equation (3-29) we achieve a fourth order equation in β :

$$
\begin{aligned}
&\beta^4(-F_{22}^2B_{21}^2 + 2F_{21}F_{22}B_{21}B_{22} + D_1D_2F_{22}B_{21} - F_{22}D_1^2B_{22} - \\
&\qquad F_{21}^2B_{22}^2 - F_{21}D_2^2B_{21} + D_1D_2F_{21}B_{22}) + \\
&\beta^3(-2E_{12}F_{22}B_{21}^2 + 2E_{11}F_{22}B_{21}B_{22} + 2F_{21}E_{12}B_{21}B_{22} + \\
&\qquad D_1D_2E_{12}B_{21} - D_1^2E_{12}B_{22} - 2E_{11}F_{21}B_{22}^2 - \\
&\qquad D_2^2E_{11}B_{21} + D_1D_2E_{11}B_{22}) + \\
&\beta^2(-E_{12}^2B_{21}^2 - 2F_{22}G_2B_{21}^2 + 2E_{11}E_{12}B_{21}B_{22} + 2F_{21}G_2B_{21}B_{22} + \\
&\qquad 2G_1F_{22}B_{21}B_{22} + D_1D_2G_2B_{21} - D_1^2G_2B_{22} - E_{11}^2B_{22}^2 - \\
&\qquad 2F_{21}G_1B_{22}^2 - D_2^2G_1B_{21} + D_1D_2G_1B_{22}) + \\
&\beta(-2E_{12}G_2B_{21}^2 + 2E_{11}G_2B_{21}B_{22} + 2G_1E_{12}B_{21}B_{22} - 2E_{11}G_1B_{22}^2) + \\
&(-G_2^2B_{21}^2 + 2G_1G_2B_{21}B_{22} - G_1^2B_{22}^2) = 0 \qquad (3\text{-}30)
\end{aligned}
$$

from which the coefficients may be determined by a substitution of Equations (3-9) through (3-18) and the values of the first kind of Chebyshev functions, Equation (2-5), in terms of ξ and ω . Since the solution of a fourth order equation is at best difficult, it is at this point a digital computer becomes a necessity.

The major problem is not the actual solution of the quartic itself, but rather the proper choice of one of the four solutions. There are two marked characteristics, however, which help in the selection. These are:

a. Complex answers to the quartic have no physical significance and may therefore be discarded as erroneous.

b. The definition of α requires that α and β be of the same sign so that p and z will be of identical sign. Using this information and that available from the Ross-Warren [2] method (Appendix I) as to compensator pole and zero location, it is found that the solution to the β quartic (Appendix II) is the largest, positive, real value.

Now entering Euqation (3-21) with this value, and evaluating the other coefficients (Appendix II) we find that:

$$\alpha = \left[- Q_1 / B_{21} \right]^{\frac{1}{2}} \qquad (3\text{-}31)$$

for in the third order case P_1 is always identically zero.

Thus, with the programing of the appropriate equations, the digital computer could give all of the values and plot the constant zeta and constant omega loci on the Parameter Plane for any desired values. With the α and β values, Figure 3-3, and the graphs of the Parameter Plane Lag, Figure 3-4, and Lead, Figure 3-5, compensation zones, a spot check of the velocity coefficient is made with:

$$K_v = \lim_{s \to 0} sG_c(s) = K_c \left(\frac{z_c}{p_c} \right)^2 \left(\frac{\prod z_i}{\prod p_i} \right) \qquad (3\text{-}32)$$

where K_c is the compensated root locus gain.

Case 1. $\zeta = .5 \quad \omega = 1.0 \qquad r = -.5 \pm j\,0.86603$

$$p_c = \beta = 0.03037 \quad \alpha = p_c/z_c = 0.36895 \quad z_c = 0.08231$$

$$K_c = \frac{(1.0)(.5^2 + .86603^2)^{\frac{1}{2}}(4.5^2 + .86603^2)^{\frac{1}{2}}(.46963^2 + .86603^2)}{(.41769^2 + .86603^2)}$$

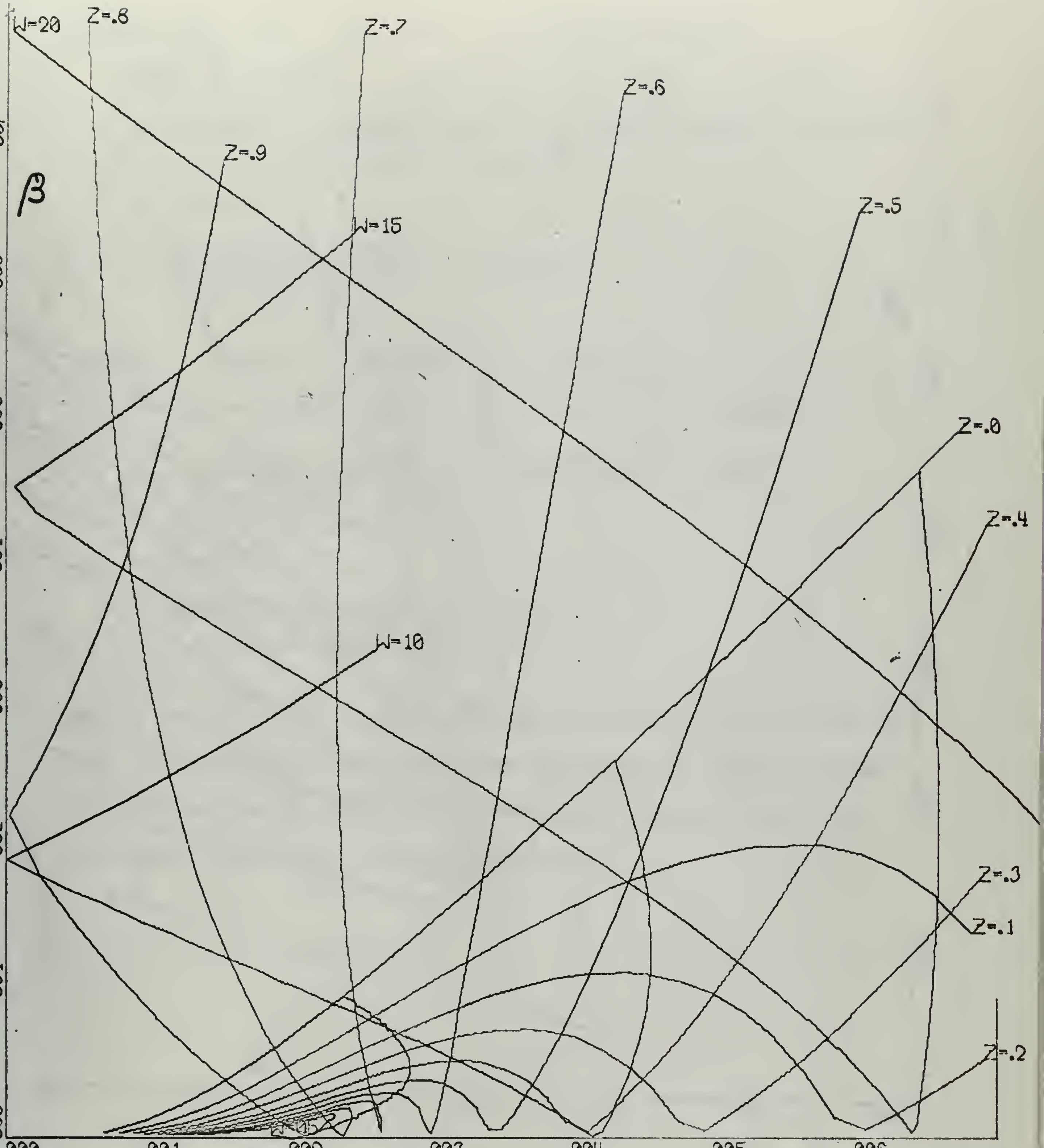
$$K_c = 4.81100$$

$$K_v = \frac{(4.81100)(.08231)^2}{(1.0)(5.0)(.03037)^2} = 7.06$$

```
   THE ALFA VALUES ARE

  ZETA      .000000     .100000     .200000     .300000     .400000
   .50 .23818E+00 .25986E+00 .27452E+00 .28300E+00 .28584E+00
  1.00 .43121E+00 .45283E+00 .45547E+00 .44130E+00 .41126E+00
  1.50 .64650E+00 .65840E+00 .62918E+00 .58083E+00 .51866E+00
  2.00 .88197E+00 .85706E+00 .80004E+00 .72553E+00 .63375E+00
  2.50 .11372E+01 .13493E+01 .22198E+01 .47367E+01 .10206E+02
  3.00 .14126E+01 .15121E+01 .17341E+01 .20955E+01 .25816E+01
  3.50 .17085E+01 .17926E+01 .19486E+01 .21789E+01 .24729E+01
  4.00 .20252E+01 .21052E+01 .22373E+01 .24211E+01 .26479E+01
  4.50 .23627E+01 .24429E+01 .25640E+01 .27247E+01 .29175E+01
  5.00 .27210E+01 .28037E+01 .29199E+01 .30679E+01 .32410E+01


   THE BETA VALUES ARE

  ZETA      .000000     .100000     .200000     .300000     .400000
   .50 .10089E+00 .86566E-01 .73874E-01 .62397E-01 .51821E-01
  1.00 .26768E+00 .18689E+00 .11578E+00 .50779E-01 .45498E-02
  1.50 .47270E+00 .17156E+00 .17741E-01 .70824E-01 .12456E+00
  2.00 .71062E+00 .51491E-01 .13997E+00 .22734E+00 .31503E+00
  2.50 .97780E+00 .47782E+01 .12213E+02 .27194E+02 .51935E+02
  3.00 .12708E+01 .28000E+01 .45954E+01 .65913E+01 .86091E+01
  3.50 .15862E+01 .28214E+01 .41652E+01 .55760E+01 .69776E+01
  4.00 .19210E+01 .30844E+01 .43124E+01 .55757E+01 .68268E+01
  4.50 .22725E+01 .34354E+01 .46439E+01 .58753E+01 .70949E+01
  5.00 .26384E+01 .38333E+01 .50638E+01 .63113E+01 .75482E+01
```

FIG 3-3

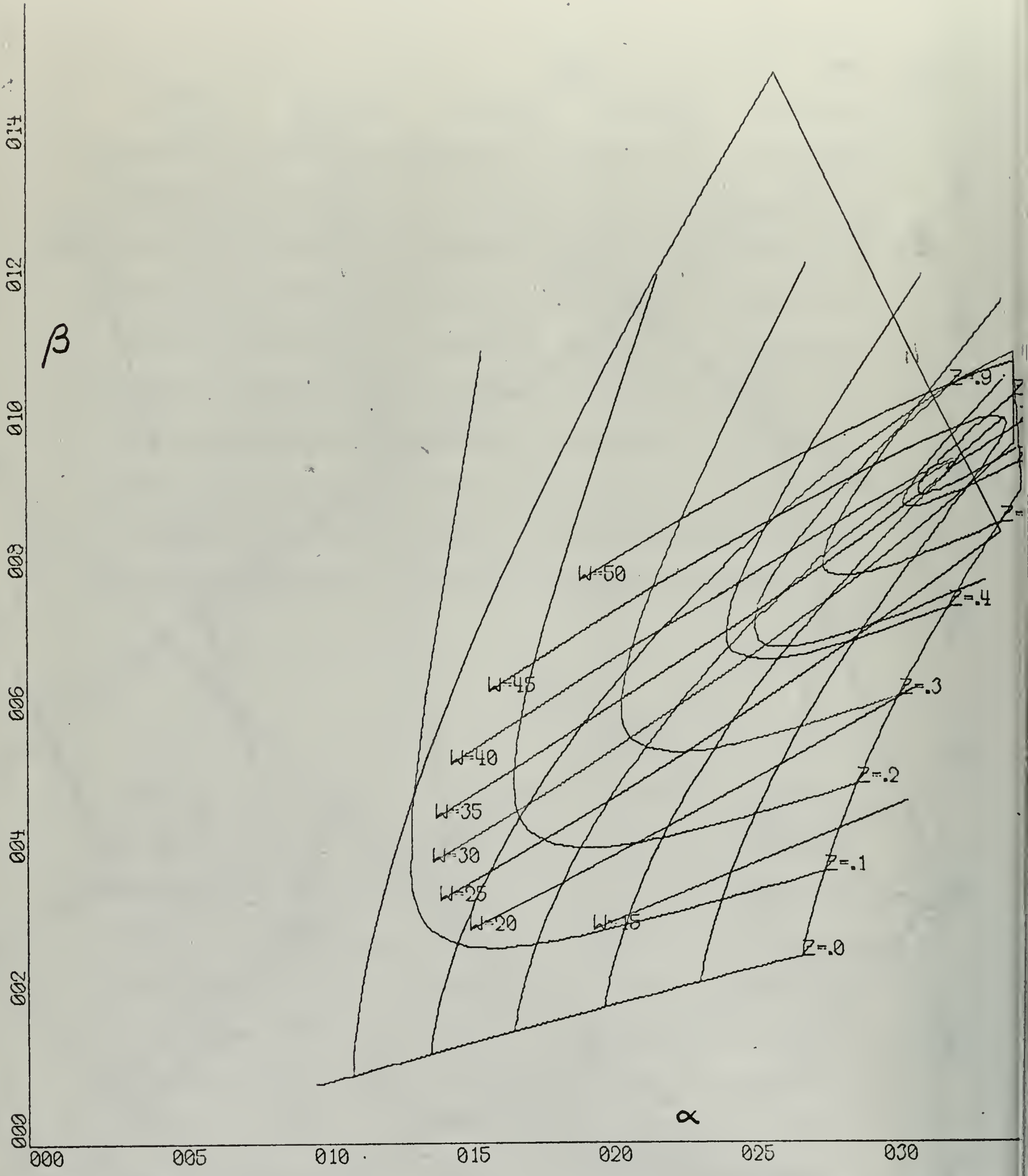
```
    .500000     .600000     .700000     .800000     .900000

 .28334E+00  .27558E+00  .26243E+00  .24341E+00  .21753E+00
 .36895E+00  .31500E+00  .24637E+00  .15610E+00  .22947E-01
 .44183E+00  .34833E+00  .23376E+00  .87524E-01  .12428E+00
 .52425E+00  .39546E+00  .24357E+00  .58854E-01  .54588E+01
 .14620E+02  .12050E+02  .79440E+01  .50060E+01  .29725E+01
 .31185E+01  .35562E+01  .36999E+01  .33953E+01  .25974E+01
 .27988E+01  .30932E+01  .32532E+01  .31366E+01  .25661E+01
 .28963E+01  .31247E+01  .32609E+01  .31848E+01  .26888E+01
 .31255E+01  .33167E+01  .34338E+01  .33730E+01  .29214E+01
 .34250E+01  .35934E+01  .36983E+01  .36495E+01  .32453E+01




    .500000     .600000     .700000     .800000     .900000

 .41891E-01  .32383E-01  .23076E-01  .13717E-01  .39634E-02
 .30370E-01  .57980E-01  .89149E-01  .12779E+00  .18641E+00
 .18046E+00  .24093E+00  .31053E+00  .39982E+00  .54396E+00
 .40506E+00  .50066E+00  .60795E+00  .74093E+00  .11957E+02
 .62975E+02  .43894E+02  .24815E+02  .13672E+02  .72851E+01
 .10318E+02  .11267E+02  .11056E+02  .95651E+01  .70152E+01
 .82452E+01  .91940E+01  .95796E+01  .91176E+01  .74993E+01
 .79904E+01  .89518E+01  .95378E+01  .94838E+01  .83371E+01
 .82489E+01  .92543E+01  .99781E+01  .10191E+02  .94146E+01
 .87333E+01  .98020E+01  .10649E+02  .11081E+02  .10659E+02
```



PARAMETER PLANE FIG 3-4

X-SCALE = 1.00E-01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

PAINE, J.A., JR. - THESIS - LAG ZONE
30.0/S(S+1.0)(S+5.0)



PARAMETER PLANE FIG 3-5

X-SCALE = 5.00E-01 UNITS/INCH.
Y-SCALE = 2.00E+00 UNITS/INCH.

PAINE, J.A., JR. - THESIS - LEAD ZONE
30.0/S(S+1.0)(S+5.0)

Case 2. $\zeta = .2 \qquad \omega = 3.0 \qquad r = -.6 \pm j\ 2.93943$

$$\beta = p_c = 4.5954 \qquad \alpha = p_c / z_c = 1.7341 \qquad z_c = 2.65002$$

$$K_c = \frac{(3.0)(.4^2 + 2.93943^2)^{\frac{1}{2}}(4.4^2 + 2.93943^2)^{\frac{1}{2}}(3.9954^2 + 2.93943^2)}{(2.05002^2 + 2.93943^2)}$$

$$K_c = 90.21638$$

$$K_v = \frac{(90.21638)(2.65002)^2}{(1.0)(5.0)(4.5954)^2} = 6.00021$$

Case 3. $\zeta = .8 \qquad \omega = 4.5 \qquad r = -3.6 \pm j\ 2.7$

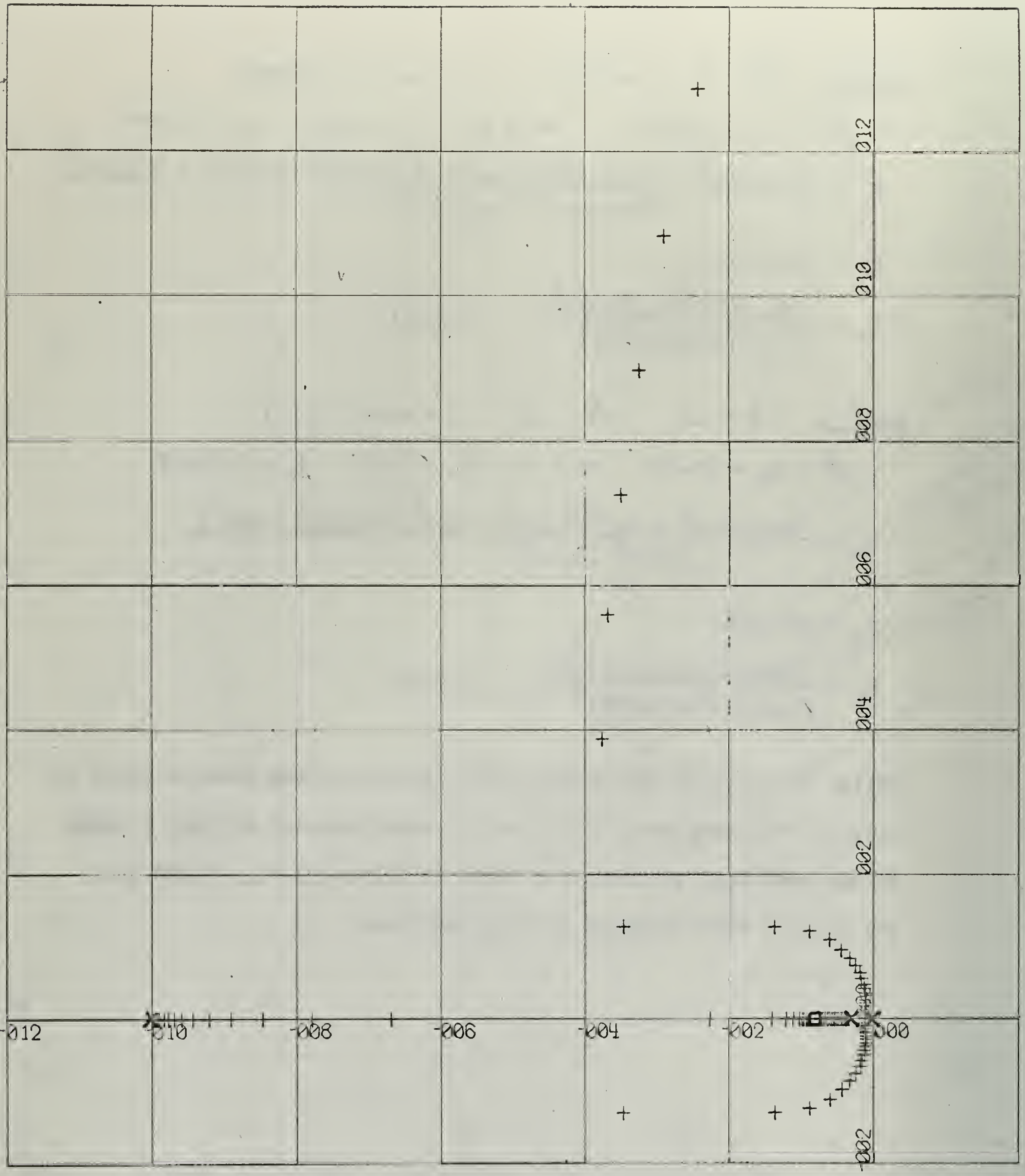
$$\beta = p_c = 10.191 \qquad \alpha = p_c / z_c = 3.373 \qquad z_c = 3.02134$$

$$K_c = \frac{(4.5)(2.6^2 + 2.7^2)^{\frac{1}{2}}(1.4^2 + 2.7^2)^{\frac{1}{2}}(6.591^2 + 2.7^2)}{(.57866^2 + 2.7^2)}$$

$$K_c = 341.3239$$

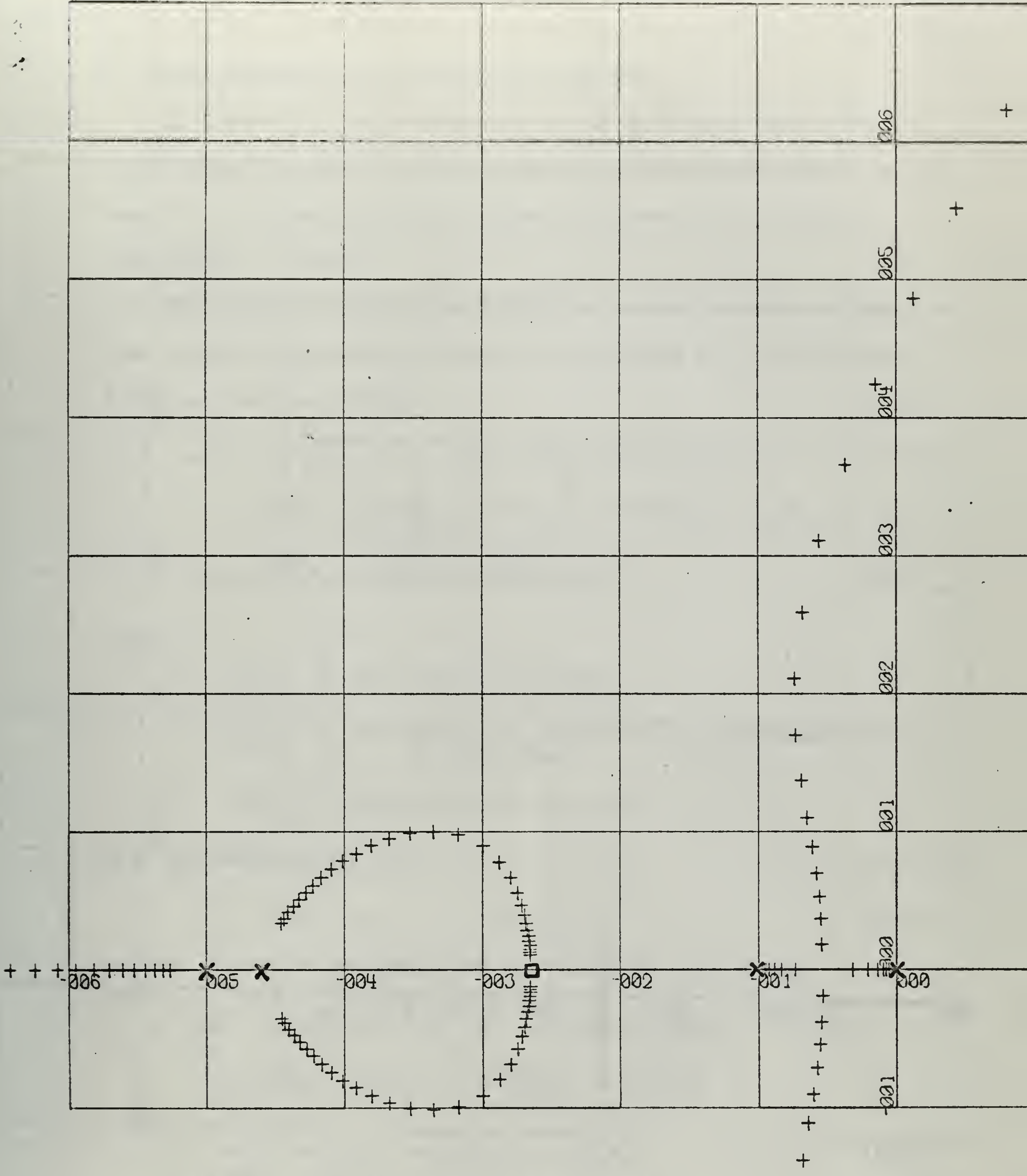
$$K_v = \frac{(341.3239)(3.02134)^2}{(1.0)(5.0)(10.191)^2} = 6.00015$$

Noting that all of the calculated K_v's are greater than or equal to 6.0, the velocity coefficient of the uncompensated system, a check of the root loci is made. As shown in Figures 3-6, 3-7 and 3-8, the desired root location is also achieved.



ROOT LOCUS FIG 3-6

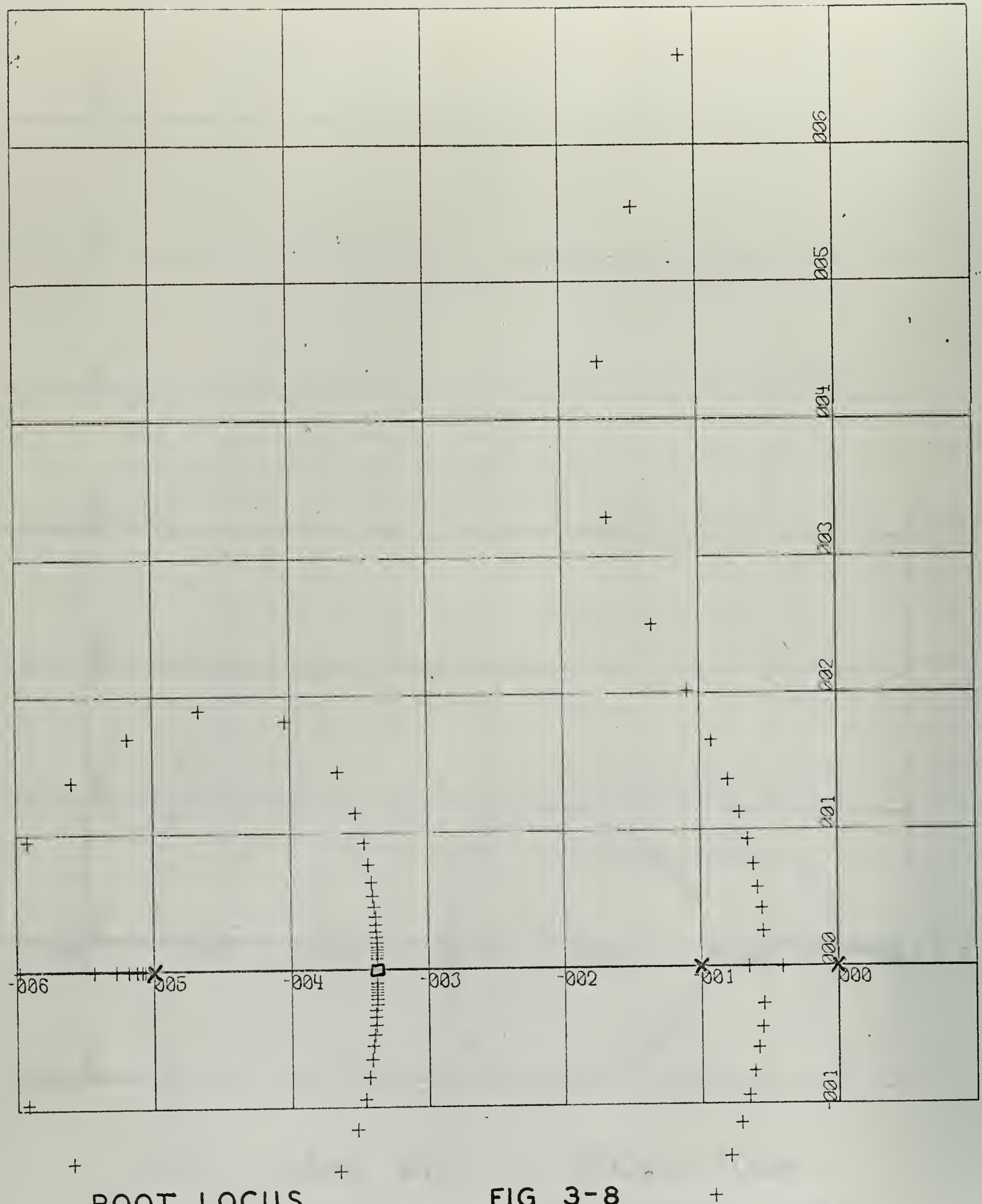
X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 2.00E-01 UNITS/INCH.
PAINE, J.A., JR. - THESIS
ZETA = 0.5 OMEGA = 1.0



ROOT LOCUS FIG 3-7

X-SCALE = 1.00E+00 UNITS/INCH.
Y-SCALE = 1.00E+00 UNITS/INCH.

PAINE, J.A., JR. - THESIS
ZETA = 0.2 OMEGA = 3.0



ROOT LOCUS FIG 3-8

X-SCALE = 1.00E+00 UNITS/INCH.
Y-SCALE = 1.00E+00 UNITS/INCH.

PAINE, J.A., JR. - THESIS
ZETA = 0.8 OMEGA = 4.5

4. Generalization of the Third Order System

In Section 3, the development of a specific type one, third order system was shown. Here a generalized development of the equations necessary to write a computer program for any third order system is shown.

Starting as before by specifying the compensated plant, (this time however in generalized terms), as in Figure 4-1, the characteristic equation is written.

$$s^5 + (2p + \sum r_i)s^4 + (p^2 + 2p\sum r_i + \sum \prod_2 r_i)s^3 + (p^2\sum r_i + 2p\sum\prod_2 r_i + \prod r_i)s^2 + (p^2\sum\prod_2 r_i + 2p\prod r_i)s + p^2\prod r_i + K(\frac{p^2}{z^2} s^2 + 2\frac{p^2}{z} s + p^2) \qquad (4\text{-}1)$$

where:

$\sum r_i \triangleq$ the sun of the roots

$\sum\prod_n r_i \triangleq$ the sum of the product of the roots taken n at a time

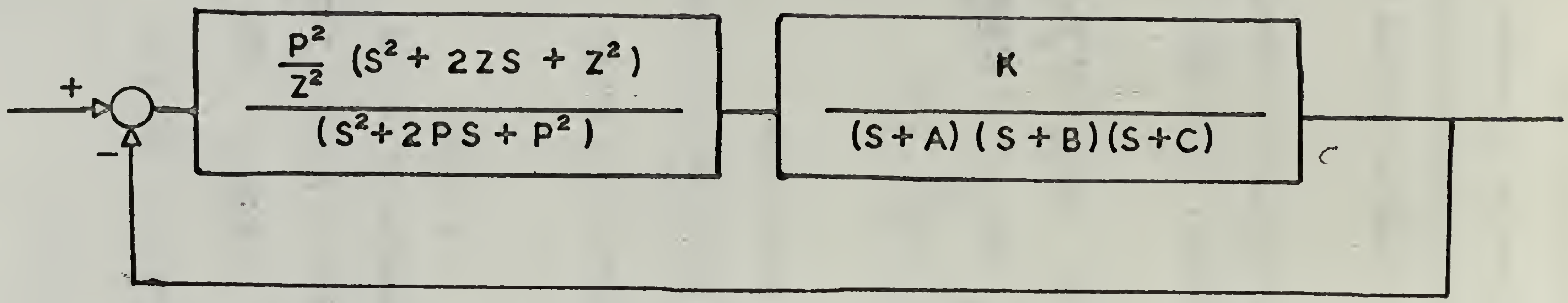
$\prod r_i \triangleq$ the product of the roots

With the substitution of:

$$\beta = p \qquad \alpha = p/z \qquad (4\text{-}2)$$

and the collection of like terms in α and β , we obtain:

$$\alpha^2(Ks^2) + \alpha\beta(2Ks) + \beta^2(s^3 + \sum r_i s^2 + \sum\prod_2 r_i s + \prod r_i + K) + \beta(2s^4 + 2\sum r_i s^3 + 2\sum\prod_2 r_i s^2 + 2\prod r_i s) + (s^5 + \sum r_i s^4 + \sum\prod_2 r_i s^3 + \prod r_i s^2) \qquad (4\text{-}3)$$



GENERALIZED COMPENSATED SYSTEM

FIG 4-1

Employing Equations (2-6) and (2-7) and defining:

$$B_{21} = K\omega^2 U_1(\xi) \tag{4-4}$$

$$B_{22} = K\omega^2 U_2(\xi) \tag{4-5}$$

$$D_1 = -2K\omega U_o(\xi) \tag{4-6}$$

$$D_2 = -2K\omega U_1(\xi) \tag{4-7}$$

$$E_{11} = 2\omega^4 U_3(\xi) - 2\sum r_i \omega^3 U_2(\xi) + 2\sum\prod_2 r_i \omega^2 U_1(\xi) - 2\prod r_i \omega U_0(\xi) \tag{4-8}$$

$$E_{12} = 2\omega^4 U_4(\xi) - 2\sum r_i \omega^3 U_3(\xi) + 2\sum\prod_2 r_i \omega^2 U_2(\xi) - 2\prod r_i \omega U_1(\xi) \tag{4-9}$$

$$F_{21} = -\omega^3 U_2(\xi) + \sum r_i \omega^2 U_1(\xi) - \sum\prod_2 r_i \omega U_o(\xi) + \left(\prod r_i + K\right) U_{-1}(\xi) \tag{4-10}$$

$$F_{22} = -\omega^3 U_3(\xi) + \sum r_i \omega^2 U_2(\xi) - \sum\prod_2 r_i \omega U_1(\xi) + \left(\prod r_i + K\right) U_o(\xi) \tag{4-11}$$

$$G_1 = -\omega^5 U_4(\xi) + \sum r_i \omega^4 U_3(\xi) - \sum\prod_2 r_i \omega^3 U_2(\xi) + \prod r_i \omega^2 U_1(\xi) \tag{4-12}$$

$$G_2 = -\omega^5 U_5(\xi) + \sum r_i \omega^4 U_4(\xi) - \sum\prod_2 r_i \omega^3 U_3(\xi) + \prod r_i \omega^2 U_2(\xi) \tag{4-13}$$

$$P_1 = \beta D_1 \tag{4-14}$$

$$P_2 = \beta D_2 \tag{4-15}$$

$$Q_1 = \beta E_{11} + \beta^2 F_{21} + G_1 \tag{4-16}$$

$$Q_2 = \beta E_{12} + \beta^2 F_{22} + G_2 \tag{4-17}$$

we find:

$$\alpha^2 B_{21} + \alpha P_1 + Q_1 = 0 \qquad (4\text{-}18)$$

$$\alpha^2 B_{22} + \alpha P_2 + Q_2 = 0 \qquad (4\text{-}19)$$

which are two non-linear algebraic equations completely generalized in terms of the uncompensated system poles and root locus gain, ζ , ω and the first kind of Chebyshev Functions. Equations (3-30) and (3-31) are now employed to obtain the solutions for α and β , using the same selection rules as before.

Several third order systems were investigated by the application of the generalized equations and the Parameter Plane curves, Figures 4-2 through 4-9, were plotted. Of these, the K/s^3 family appears the most interesting. Further investigation of three of the curves in this family, Figures 4-2, 4-3 and 4-4 shows that there is a relationship between K, the root locus gain, α and β .

These relations are:

a. Choose a point on the $1/s^3$ α -β plane.

b. Zeta reads directly.

c. Determine the actual omega at that point by multiplying the value read by the cube root of the uncompensated system gain.

d. Read the value of α directly from the point chosen.

e. Read the value of β from the point chosen.

f. Obtain the true value of β by multiplying this value by the cube root of the uncompensated system gain.

By this method, the values of α and β may be determined for

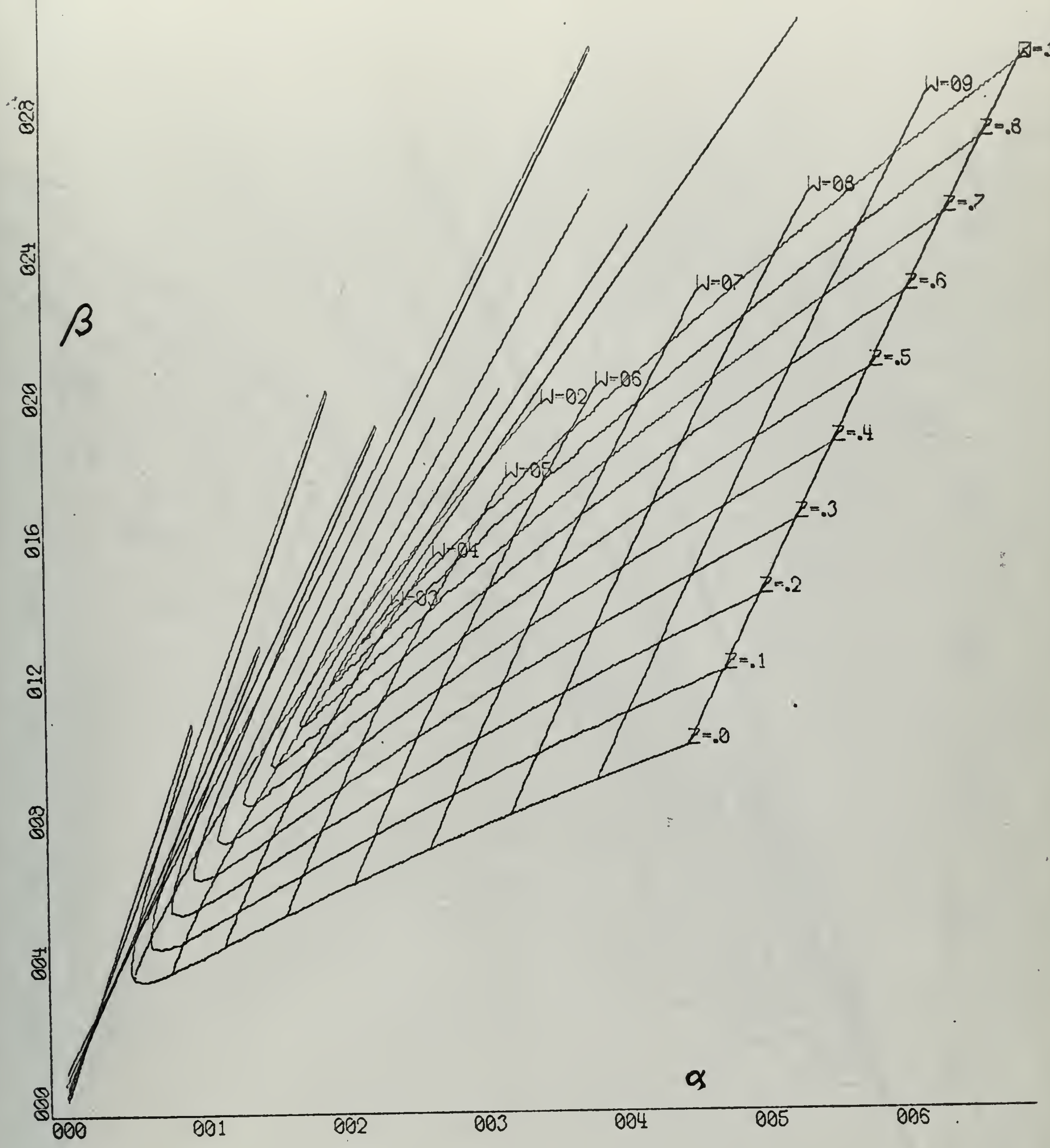


FIG 4-2

X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. 1/S**3
LEAD ZONE

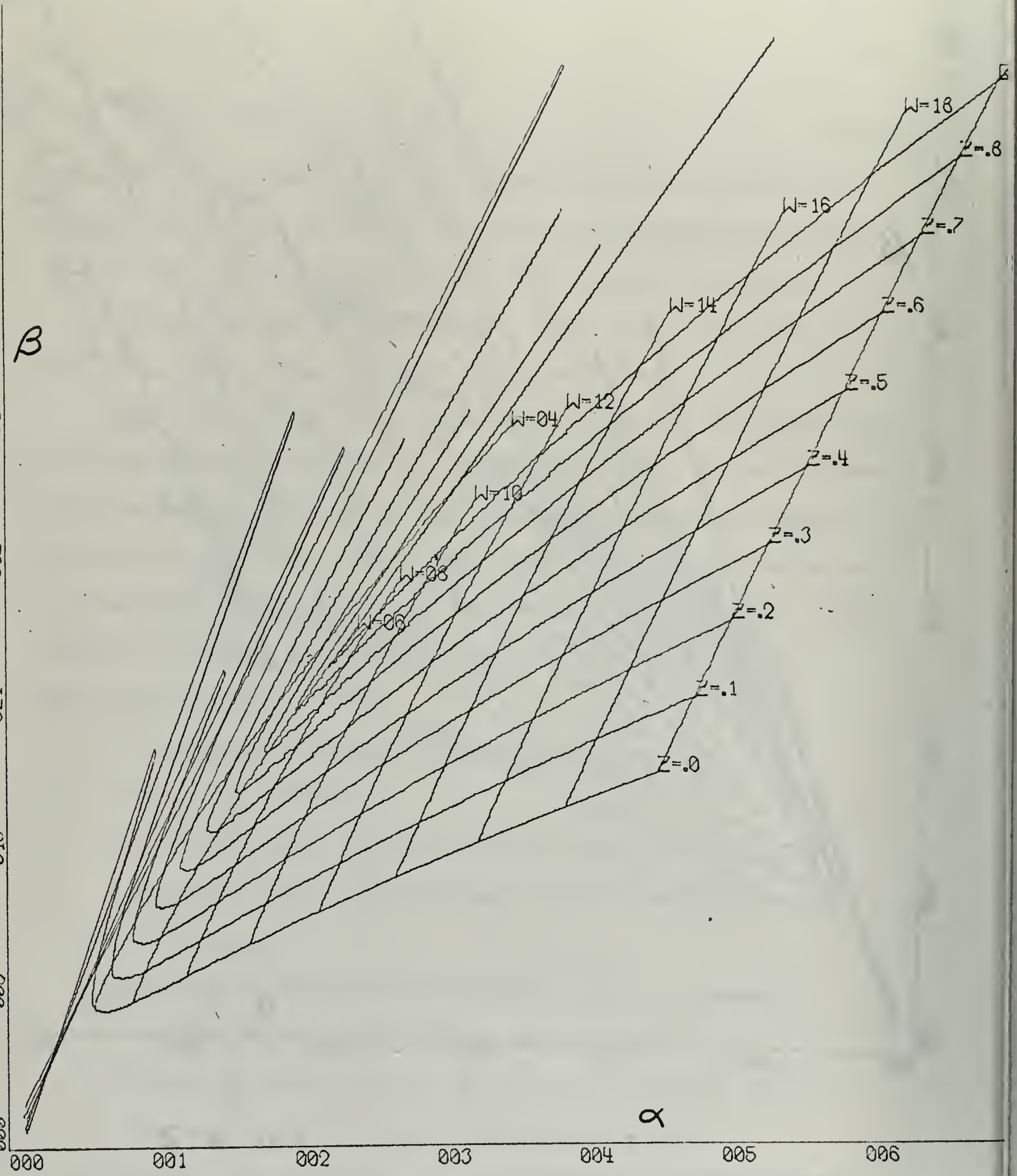


FIG 4-3

X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 8.00E+00 UNITS/INCH.

PAINE, J.A., JR.   LEAD ZONE
8.0 / S**3

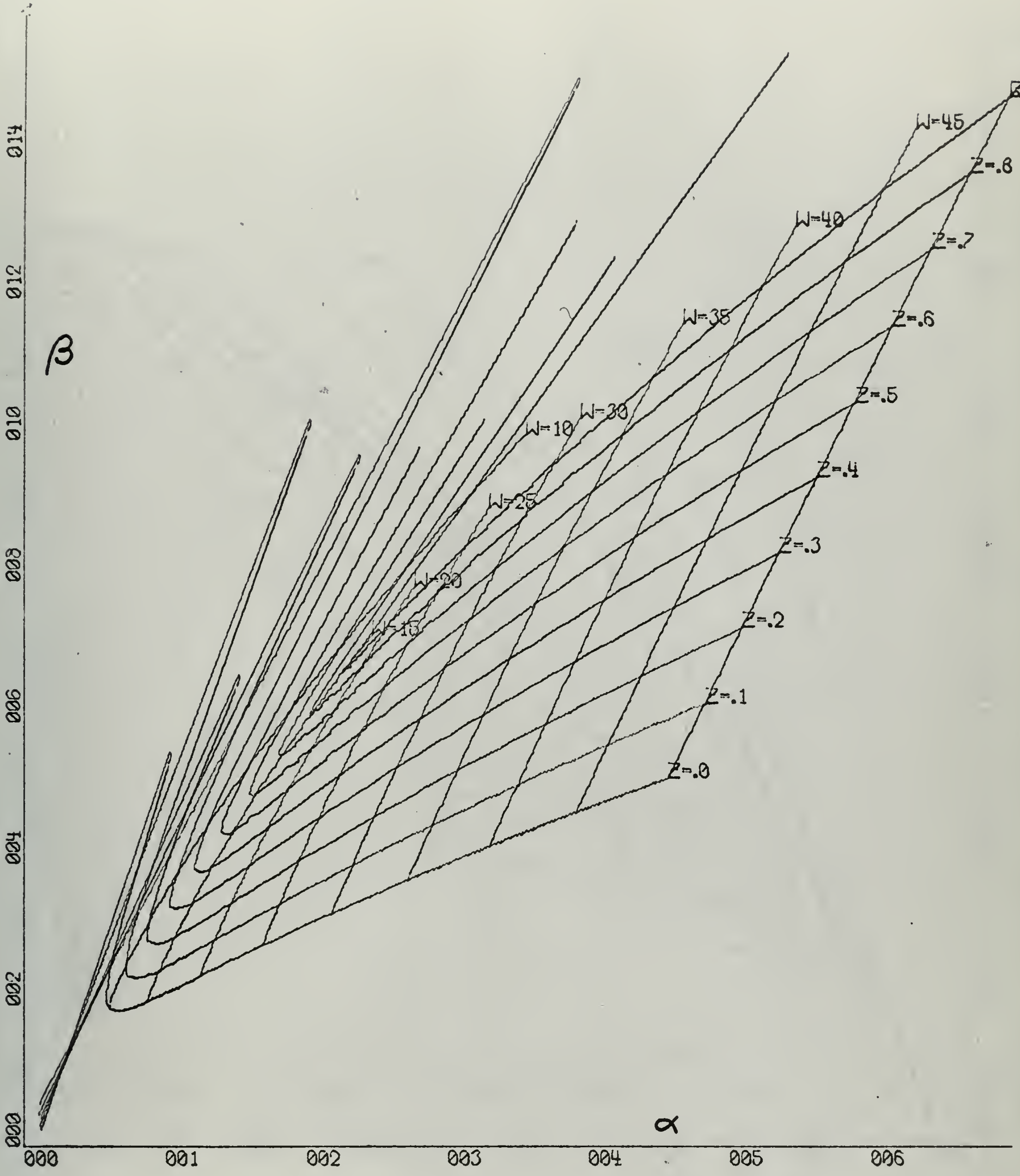


FIG 4-4

X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 2.00E+01 UNITS/INCH.
PAINE, J.A., JR. LEAD ZONE
125.0 / S**3

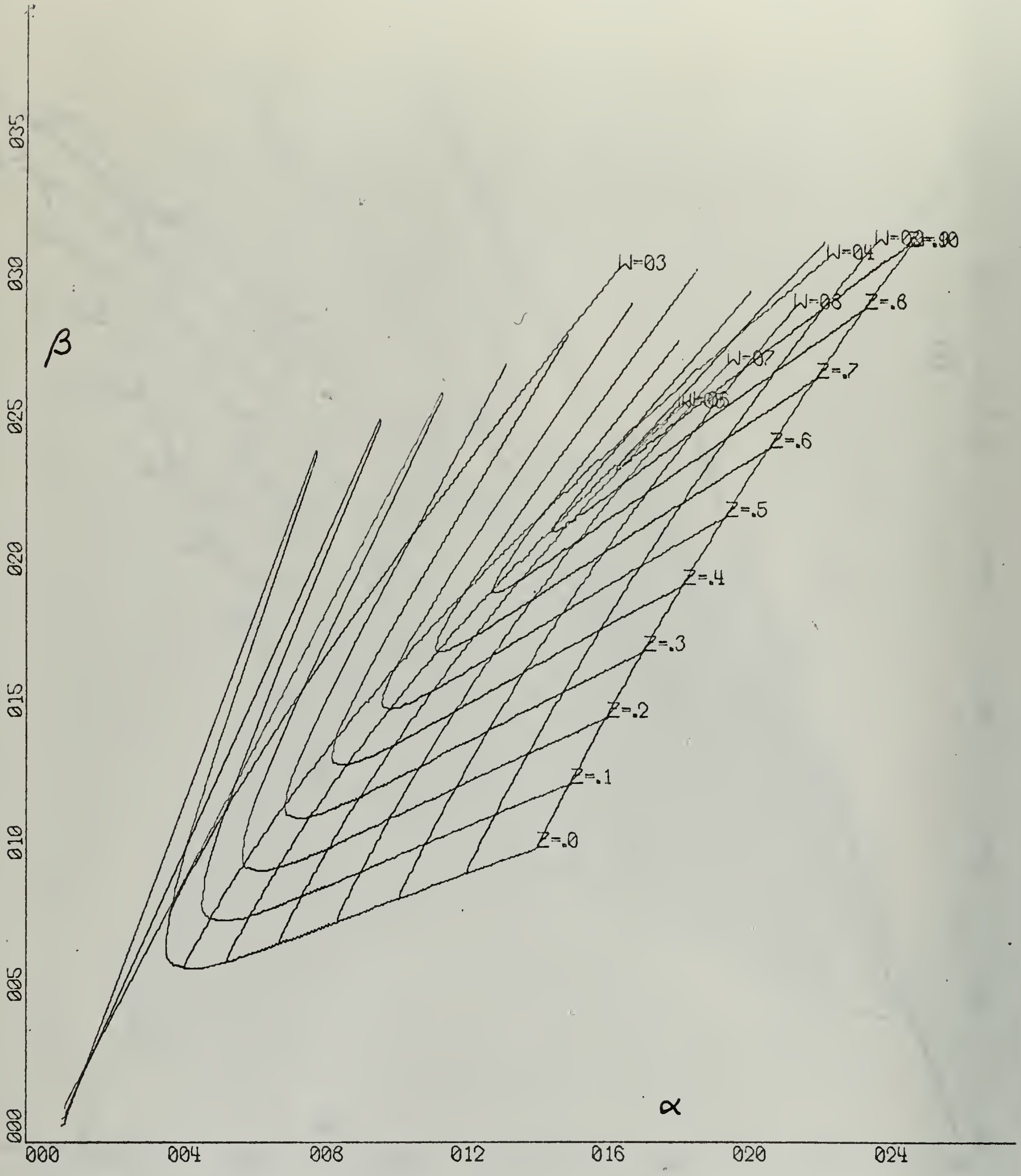


FIG 4-5

X-SCALE = 4.00E+00 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.

PAINE, J.A., JR.   LEAD ZONE
10/S**2(S+1)

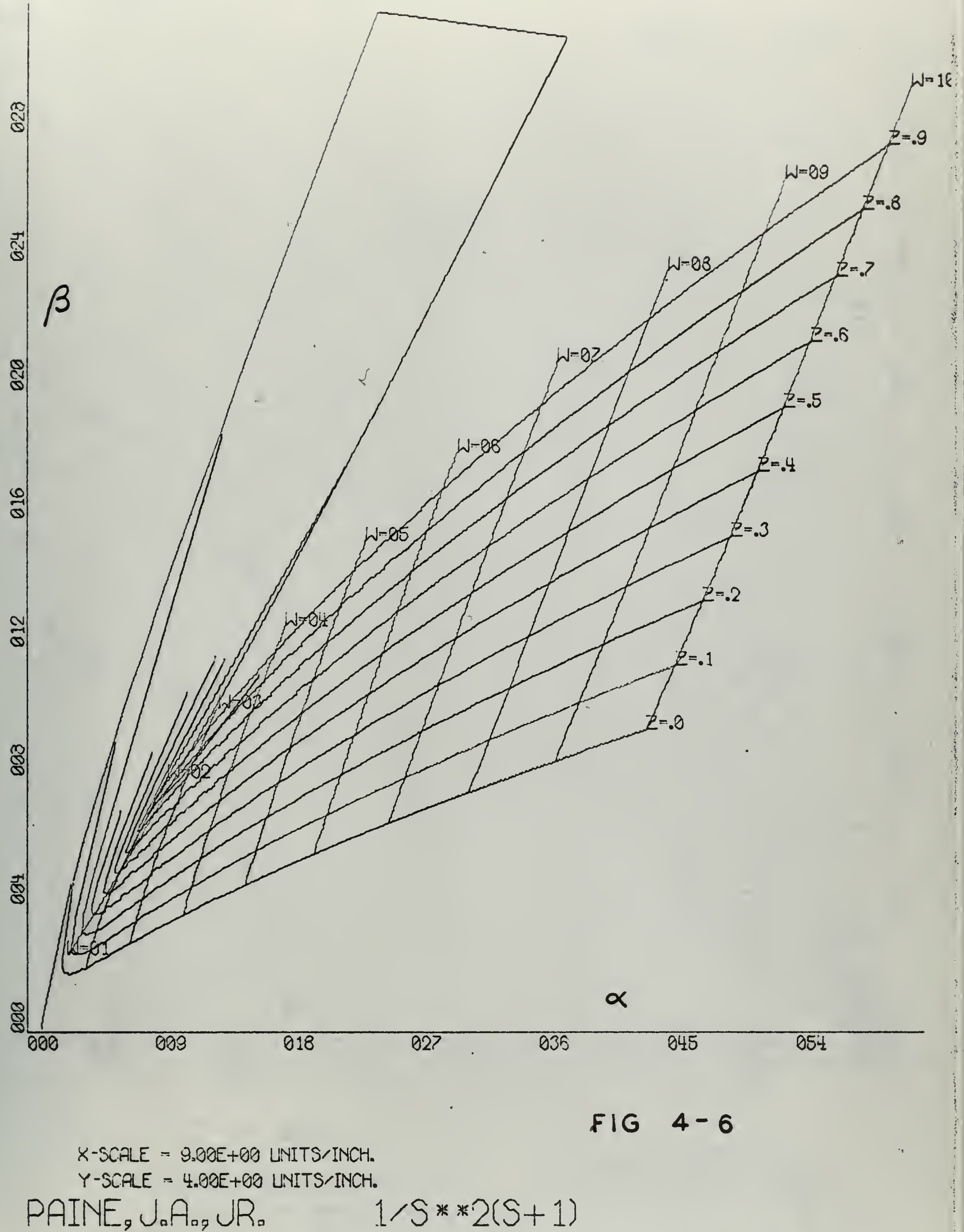


FIG 4-6

X-SCALE = 9.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. 1/S**2(S+1)
LEAD ZONE

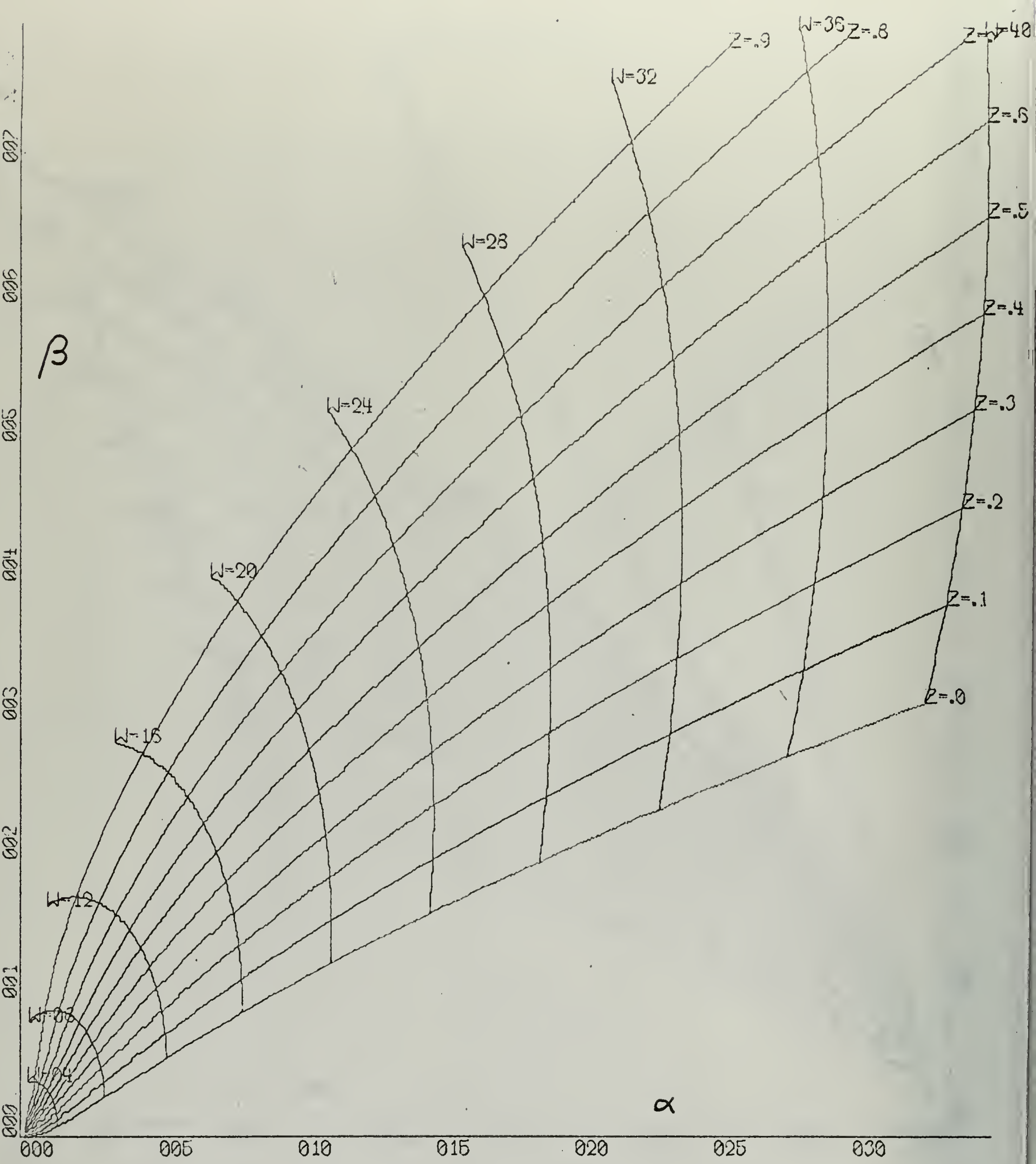


FIG 4-7

X-SCALE = 5.00E+01 UNITS/INCH.
Y-SCALE = 1.00E+01 UNITS/INCH.

PAINE, J.A., JR. 1/S**2(S+10)
LEAD ZONE

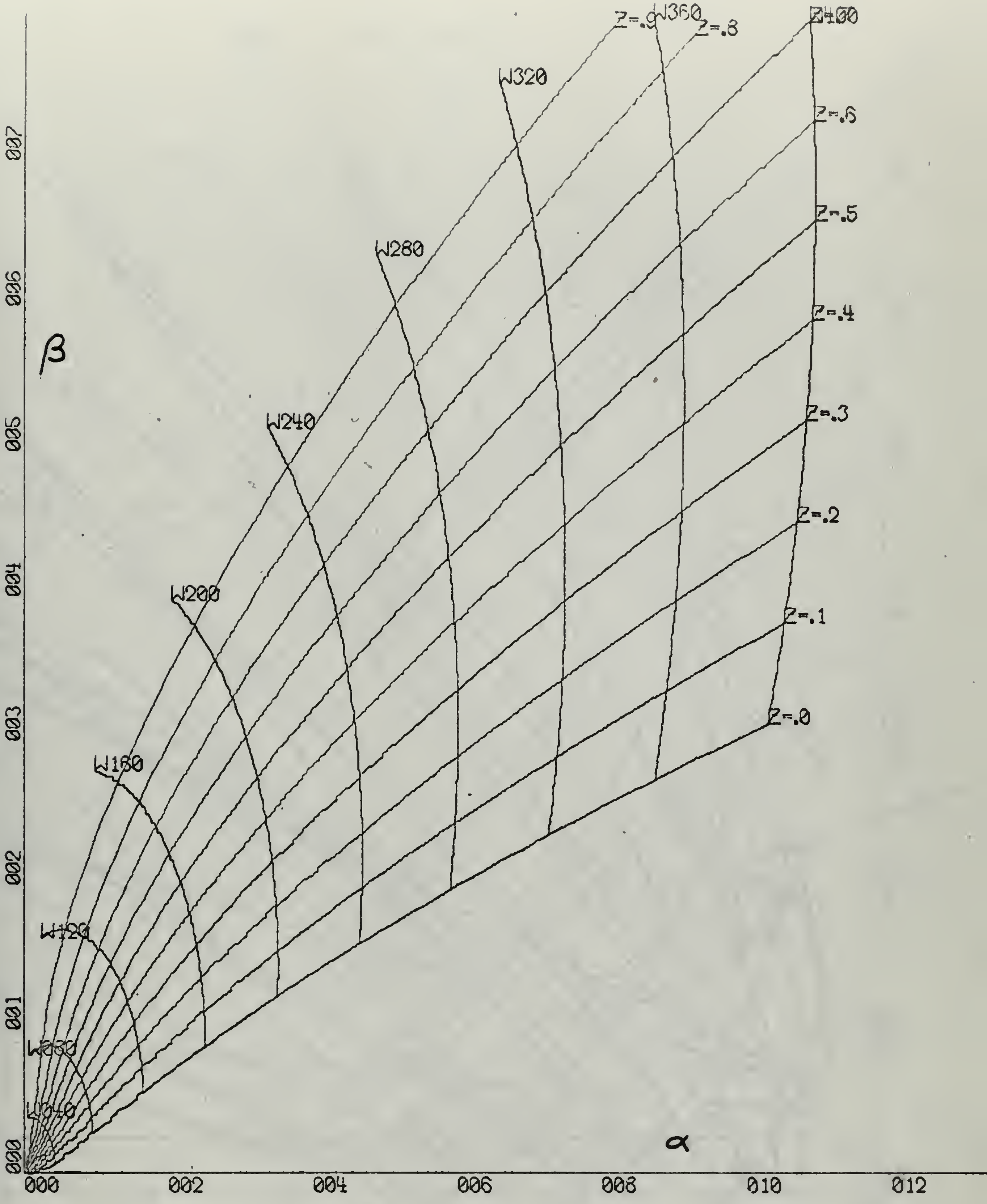


FIG 4-8

X-SCALE = 2.00E+03 UNITS/INCH.
Y-SCALE = 1.00E+02 UNITS/INCH.

PAINE, J.A., JR. 1/S**2(S+100)
LEAD ZONE

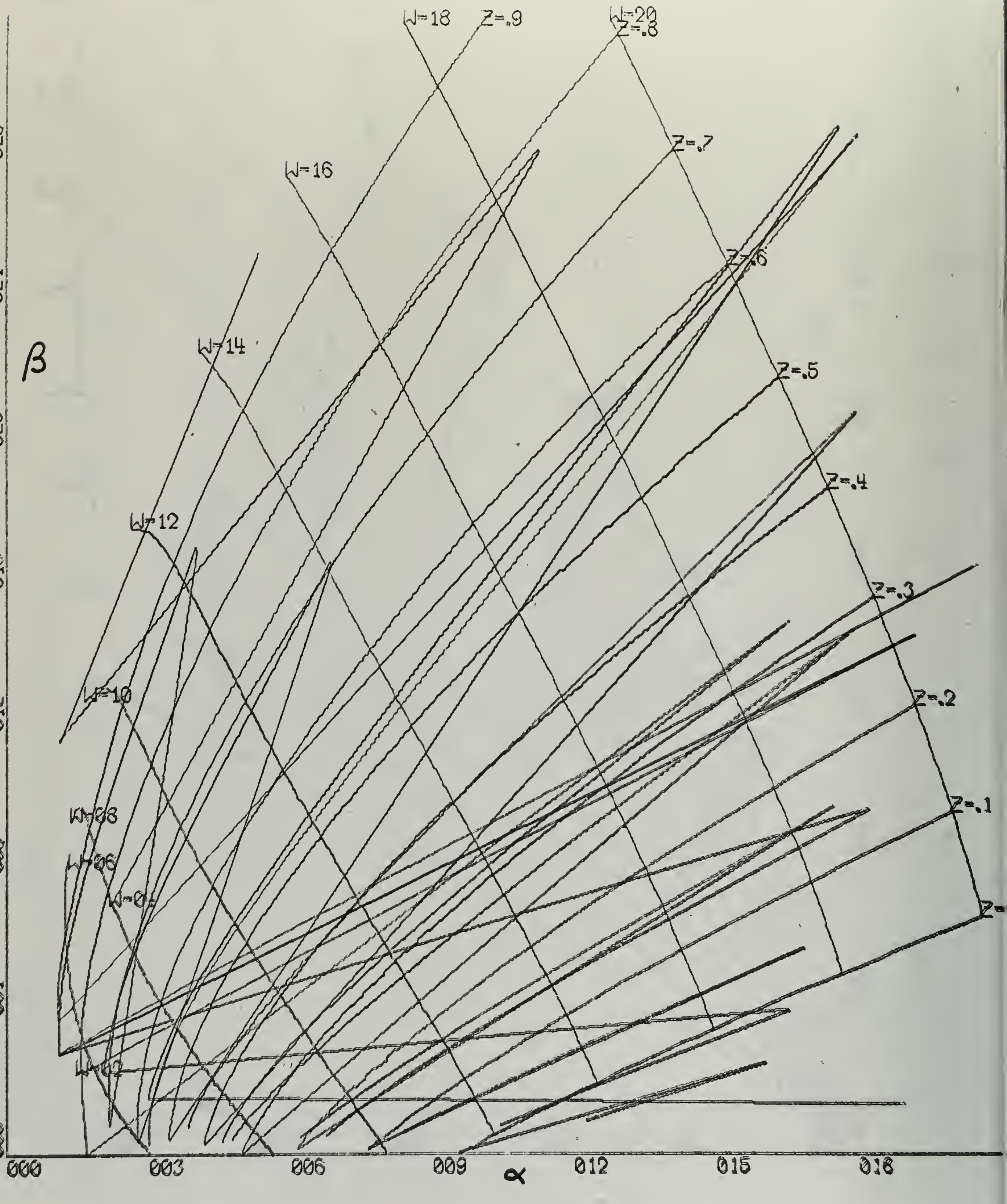


FIG 4-9

X-SCALE = 3.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. LEAD ZONE
25.0 / (S+5.0)(S+5.0)(S+10.0)

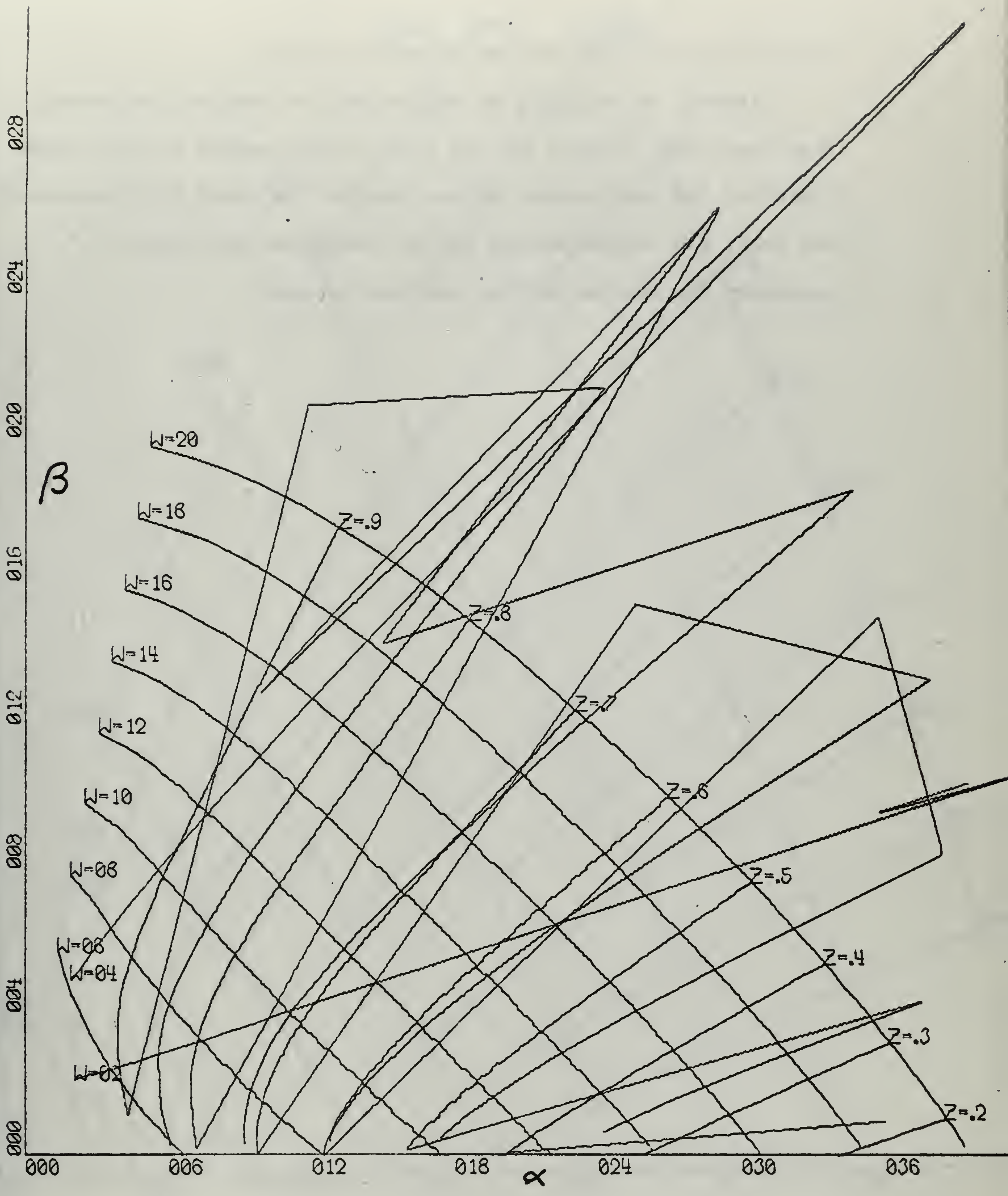


FIG 4-10

X-SCALE = 6.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. LEAD ZONE
25.0 / (S+5.0)(S+5.0)(S+100.0)

all K/s^3 systems from the one universal curve.

Although the majority of the curves are easily interpreted, there are some, Figures 4-9 and 4-10, which require careful correlation of the data points and the curves. As shown in Figures 4-11 and 4-12, the correct curves may be picked out and redrawn if necessary for further clarity and ease of use.

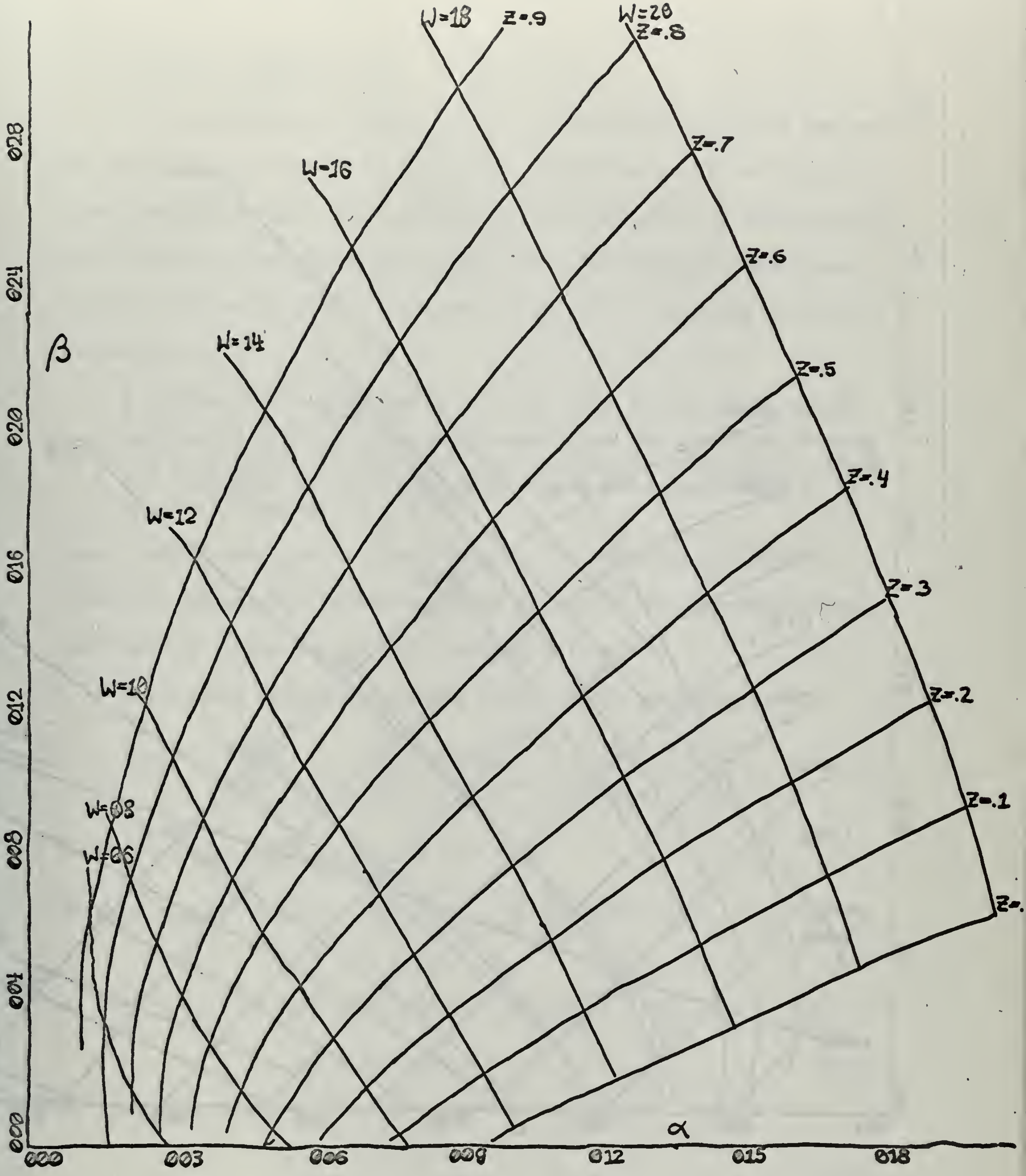


FIG 4-11

X-SCALE = 3.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR.   LEAD ZONE
25.0 / (S+5.0)(S+5.0)(S+10.0)

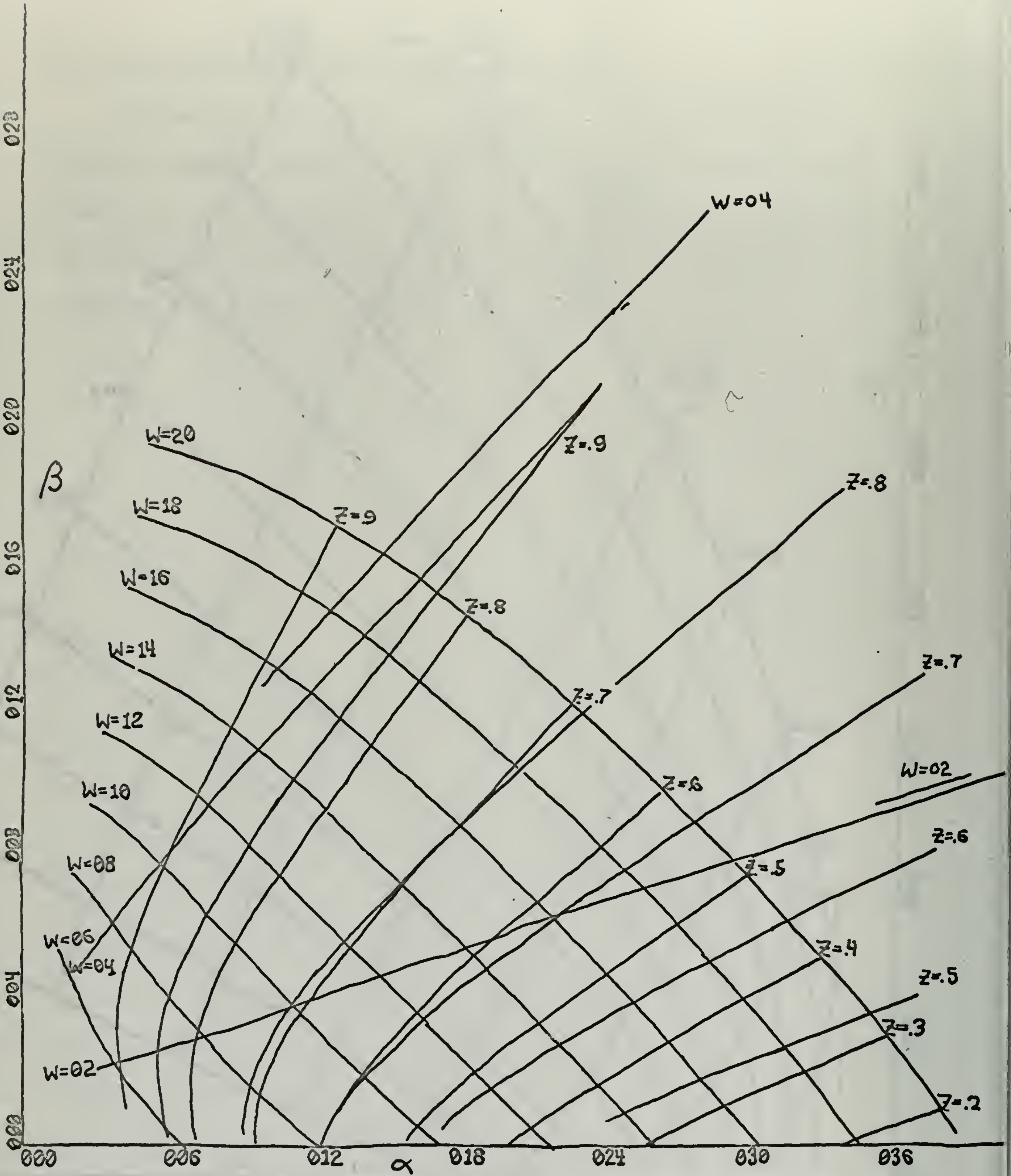


FIG 4-12

X-SCALE = 6.00E+00 UNITS/INCH
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR.   LEAD ZONE
25.0 / (S+5.0)(S+5.0)(S+100.0)

5. Bandwidth Curves on the $\alpha - \beta$ Plane

In many instances, there is also a bandwidth criterion imposed on the engineer as well as an optimal operating point for the plant under consideration. With this in mind, equations for the plotting of constant bandwidth curves on the $\alpha - \beta$ plane are developed. For the purpose of this development a constant bandwidth curve will be defined as:

> A constant bandwidth curve for $G(j\omega_b) = M$ is a curve drawn upon the parameter plane which specifies the relation between the parameters necessary if the transfer function $G(s)$, which is a function of the parameters, is to have magnitude M at the real frequency ω_b. [5]

Once these curves are obtained they may be superimposed on the parameter plane thus indicating what values of the parameters are necessary in order to meet the specifications.

Taking the rational transfer function and defining it:

$$G(s) = \frac{P(s)}{Q(s)} = \frac{p_m s^m + p_{m-1} s^{m-1} + \dots p_1 s + p_o}{q_n s^n + q_{n-1} s^{n-1} + \dots q_1 s + q_o} \qquad (5\text{-}1)$$

where the p_m's and q_n's are of the form:

$$p_u = g_u \alpha^2 + h_u \alpha + i_u \alpha\beta + j_u \beta + k_u \beta^2 + l_u$$

$$u = 0,1,2,\dots,m \qquad (5\text{-}2)$$

$$q_v = a_v \alpha^2 + b_v \alpha + c_v \alpha\beta + d_v \beta + e_v \beta^2 + f_v$$

$$v = 0,1,2,\dots,n \qquad (5\text{-}3)$$

we may write:

$$G(s) = \frac{\sum_{u=0}^{m} p_u s^u}{\sum_{v=0}^{n} q_v s^v} \qquad (5\text{-}4)$$

Employing Equation (5-4) we write in the parameterized form the generalized compensated third order transfer function:

$$G(s) = \frac{P(s)}{Q(s)} \tag{5-5}$$

where:

$$P(s) = \alpha^2 Ks^2 + 2\alpha\beta Ks + \beta^2 K \tag{5-6}$$

and:

$$\begin{aligned} Q(s) = {} & \beta^2\left[s^3 + \sum\prod_1 r_i s^2 + \sum\prod_2 r_i s + \sum\prod_3 r_i\right] + \\ & \beta\left[2s^4 + 2\sum\prod_1 r_i s^3 + 2\sum\prod_2 r_i s^2 + 2\sum\prod_3 r_i s\right] + \\ & \left[s^5 + \sum\prod_1 r_i s^4 + \sum\prod_2 r_i s^3\right] + \sum\prod_3 r_i s^2 \end{aligned} \tag{5-7}$$

Making the definitions:

$$A_r = \sum_{\substack{v=0 \\ \text{even}}}^{n} (-1)^{\frac{1}{2}v}\omega_b^v a_v \text{ ; etc. for } B_r, C_r, D_r, E_r, F_r \tag{5-8}$$

$$A_i = \sum_{\substack{v=0 \\ \text{odd}}}^{n} (-1)^{\frac{1}{2}(v-1)}\omega_b^v a_v\text{; etc. for } B_i, C_i, D_i, E_i, F_i \tag{5-9}$$

$$G_r = \sum_{\substack{u=0 \\ \text{even}}}^{m} (-1)^{\frac{1}{2}u}\omega_b^u g_u \text{ ; etc. for } H_r, I_r, J_r, K_r, L_r \tag{5-10}$$

$$G_i = \sum_{\substack{u=0 \\ \text{odd}}}^{m} (-1)^{\frac{1}{2}(u-1)}\omega_b^u g_u\text{; etc. for } H_i, I_i, J_i, K_i, L_i \tag{5-11}$$

and substituting in Equation (5-5), we write:

$$G(j\omega_b) = \frac{(\alpha^2 G_r + K_r) + j(\alpha\beta I_i)}{(\beta^2 D_r + \beta E_r + F_r) + j(\beta^2 D_i + \beta E_i + F_i)} \tag{5-12}$$

Setting the magnitude of $G(j\omega_b) = M$ we find:

$$M^2 = \left|G(j\omega_b)\right|^2 = \frac{(\alpha^2 G_r + K_r)^2 + (\alpha\beta I_i)^2}{(\beta^2 D_r + \beta E_r + F_r)^2 + (\beta^2 D_i + \beta E_i + F_i)^2} \tag{5-13}$$

Manipulating Equation (5-13) algebraically we find that:

$$\phi(\alpha,\beta) - M^2\theta(\alpha,\beta) = 0 \tag{5-14}$$

where:

$$\phi(\alpha,\beta) = \alpha^2 G_r^2 + 2\alpha^2 K_r G_r + K_r^2 + \alpha^2\beta^2 I_i^2 \tag{5-15}$$

$$\begin{aligned}\theta(\alpha,\beta) = {} & \beta^4 D_r^2 + 2\beta^3 D_r E_r + 2\beta^2 D_r F_r + \beta^2 E_r^2 + \\ & 2\beta E_r F_r + F_r^2 + \beta^4 D_i^2 + \beta^2 E_i^2 + F_i^2 + \\ & 2\beta^3 E_i D_i + 2\beta^2 D_i F_i + 2\beta E_i F_i\end{aligned} \tag{5-16}$$

Substituting Equations (5-15) and (5-16) in Equation (5-14) and defining:

$$P_1 = D_r^2 + D_i^2 \tag{5-17}$$

$$Q_1 = 2D_r E_r + 2E_i D_i \tag{5-18}$$

$$R_1 = 2D_r F_r + E_r^2 + E_i^2 + 2D_i F_i \tag{5-19}$$

$$R_2 = \alpha^2 I_i^2 \tag{5-20}$$

$$V_1 = 2E_r F_r + 2E_i F_i \tag{5-21}$$

$$W_1 = F_r^2 + F_i^2 \tag{5-22}$$

$$W_2 = \alpha^4 G_r^2 + 2\alpha^2 K_r G_r + K_r^2 \tag{5-23}$$

we can write:

$$M^2P_1\beta^4 + M^2Q_1\beta^3 + (M^2R_1 - R_2)\beta^2 + M^2V_1\beta + (M^2W_1 - W_2) = 0 \qquad (5\text{-}24)$$

Since the Parameter Plane for compensation purposes has already been determined it is now a matter of taking the computed α values and substituting them along with a constant value of omega and M into Equation (5-24) and then solving the β quartic. This has as its solution the largest, real and positive value of the four roots as before.

Examples of these curves, drawn so as to enable their use as overlays on Figures 4-2 through 4-10 are shown in Figures 5-1 through 5-9.

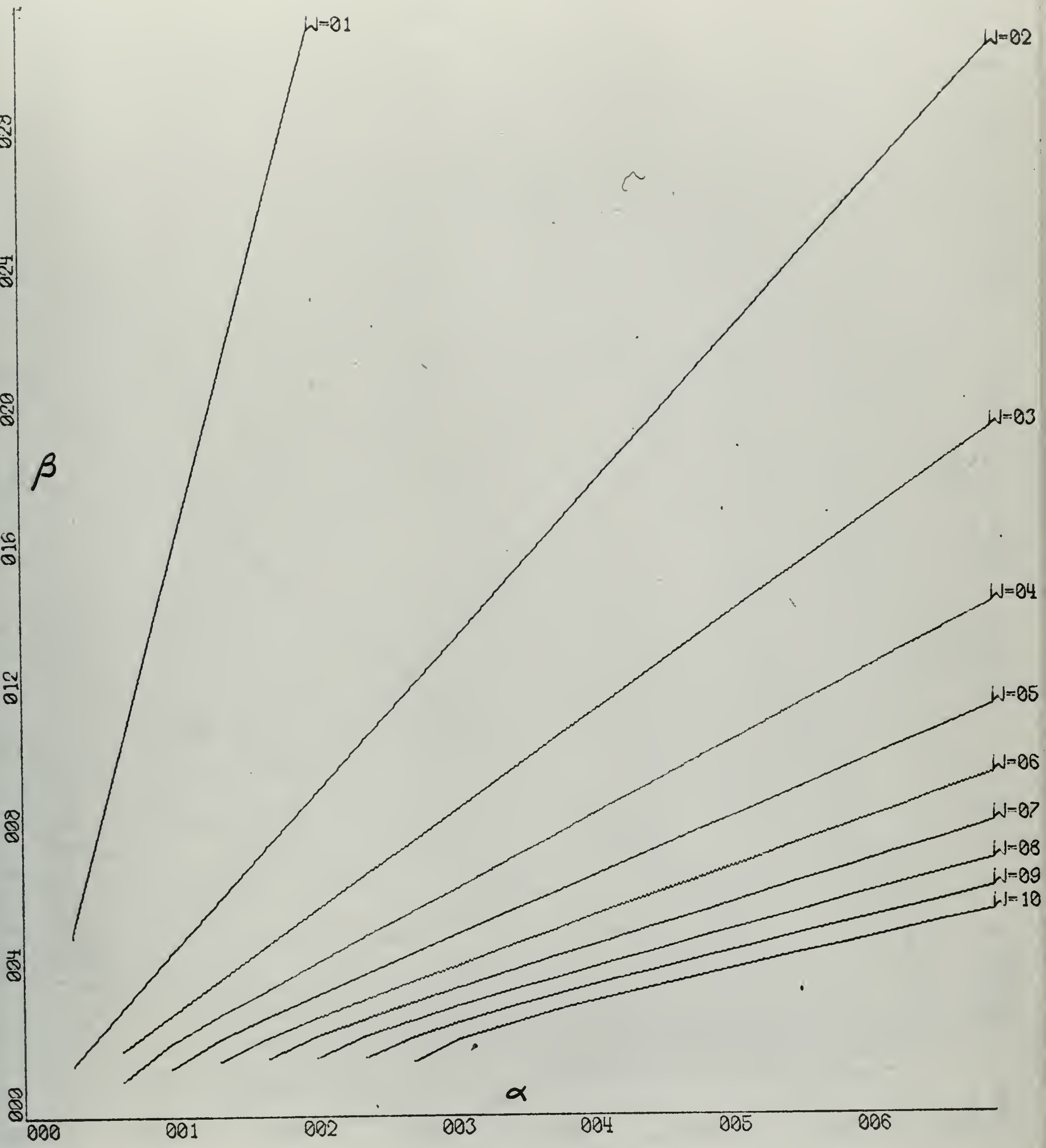


FIG 5-1

X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. 1/S**3
CONSTANT BANDWIDTH

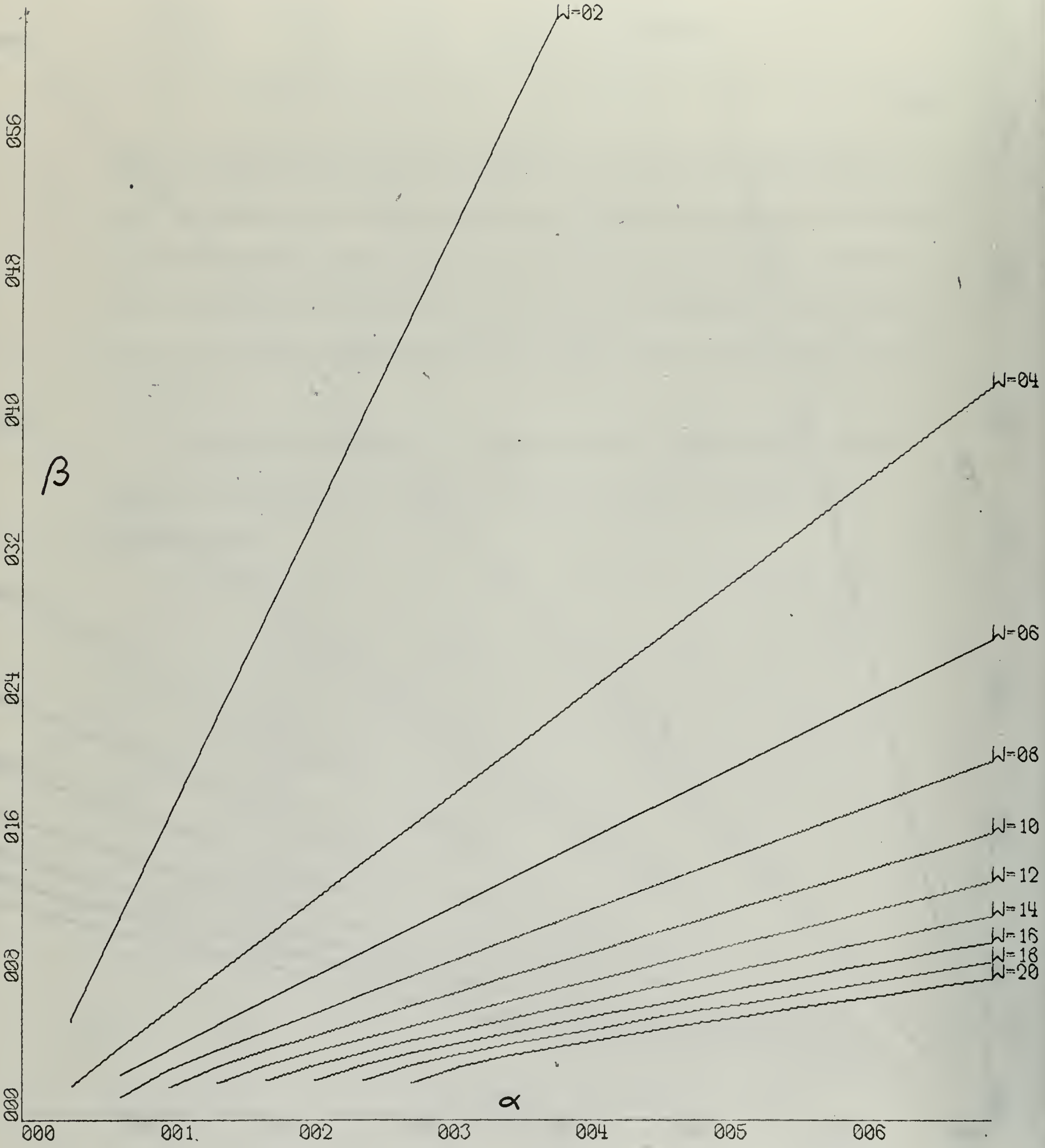


FIG 5-2

X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 8.00E+00 UNITS/INCH.

PAINE, J.A., JR. CONSTANT BANDWIDTH
8.0 / S**3

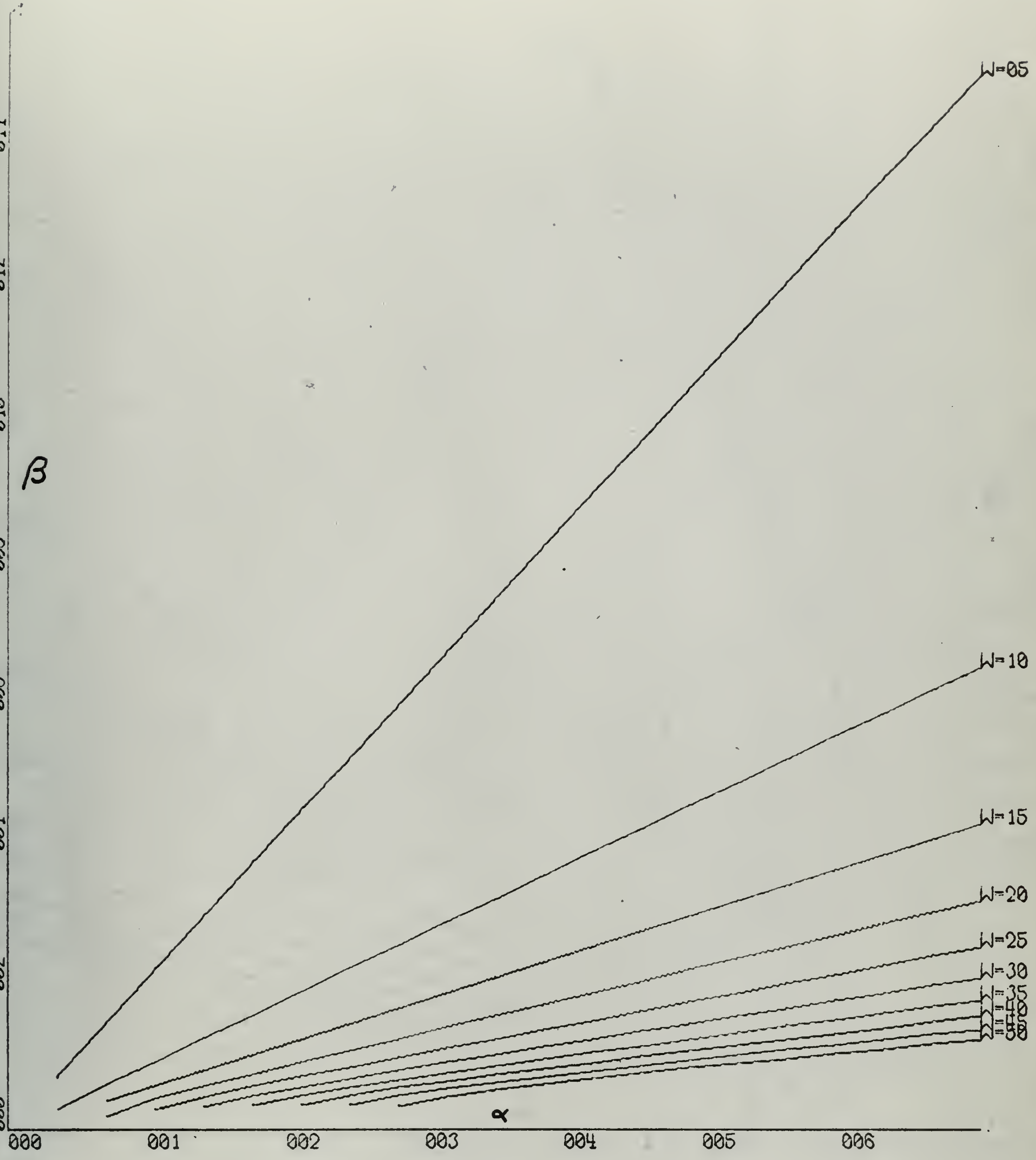


FIG 5-3

X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 2.00E+01 UNITS/INCH.

PAINE, J.A., JR. CONSTANT BANDWIDTH
125.0 / S**3

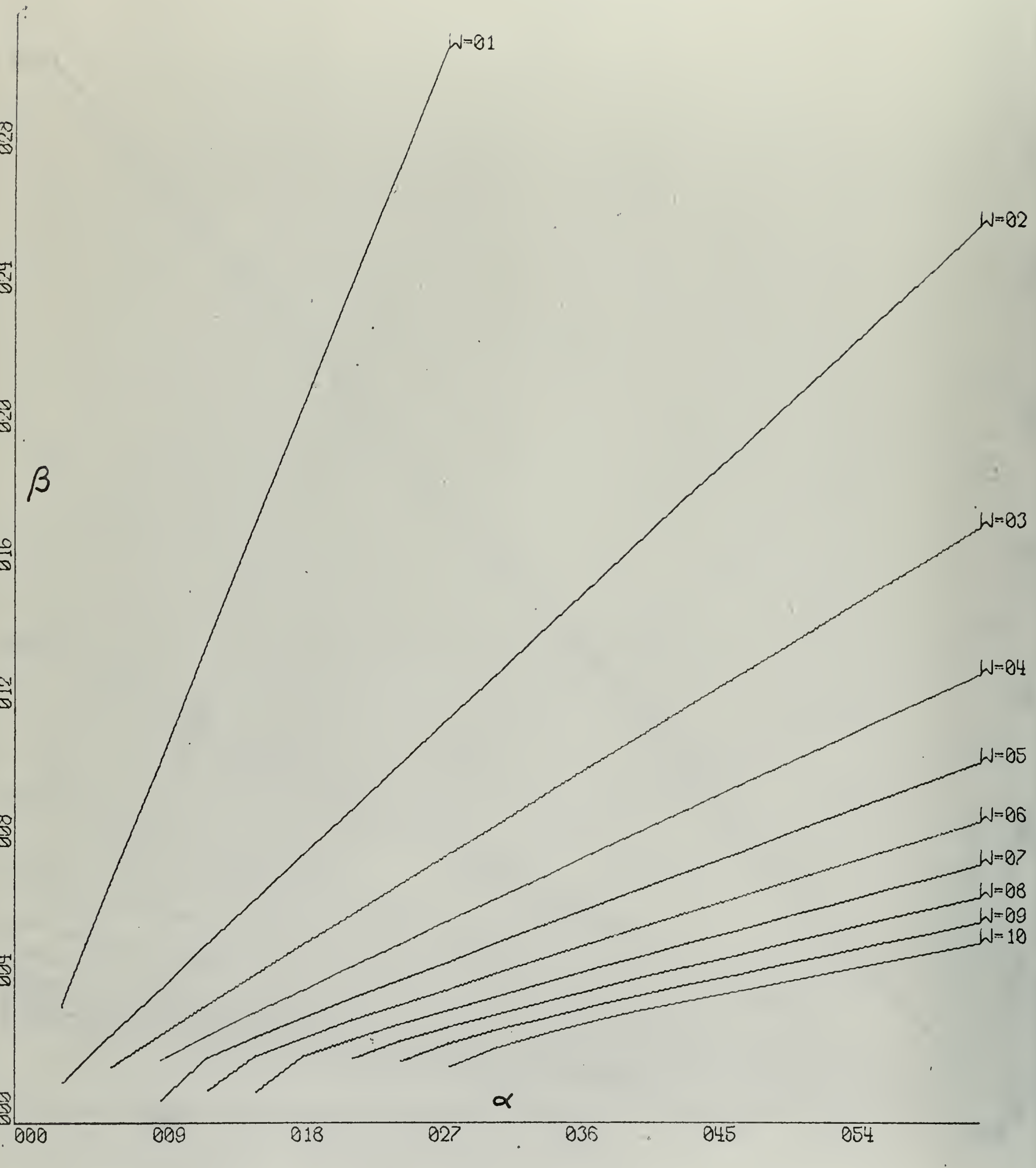


FIG 5-4

X-SCALE = 9.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. 1/S**2(S+1)
CONSTANT BANDWIDTH

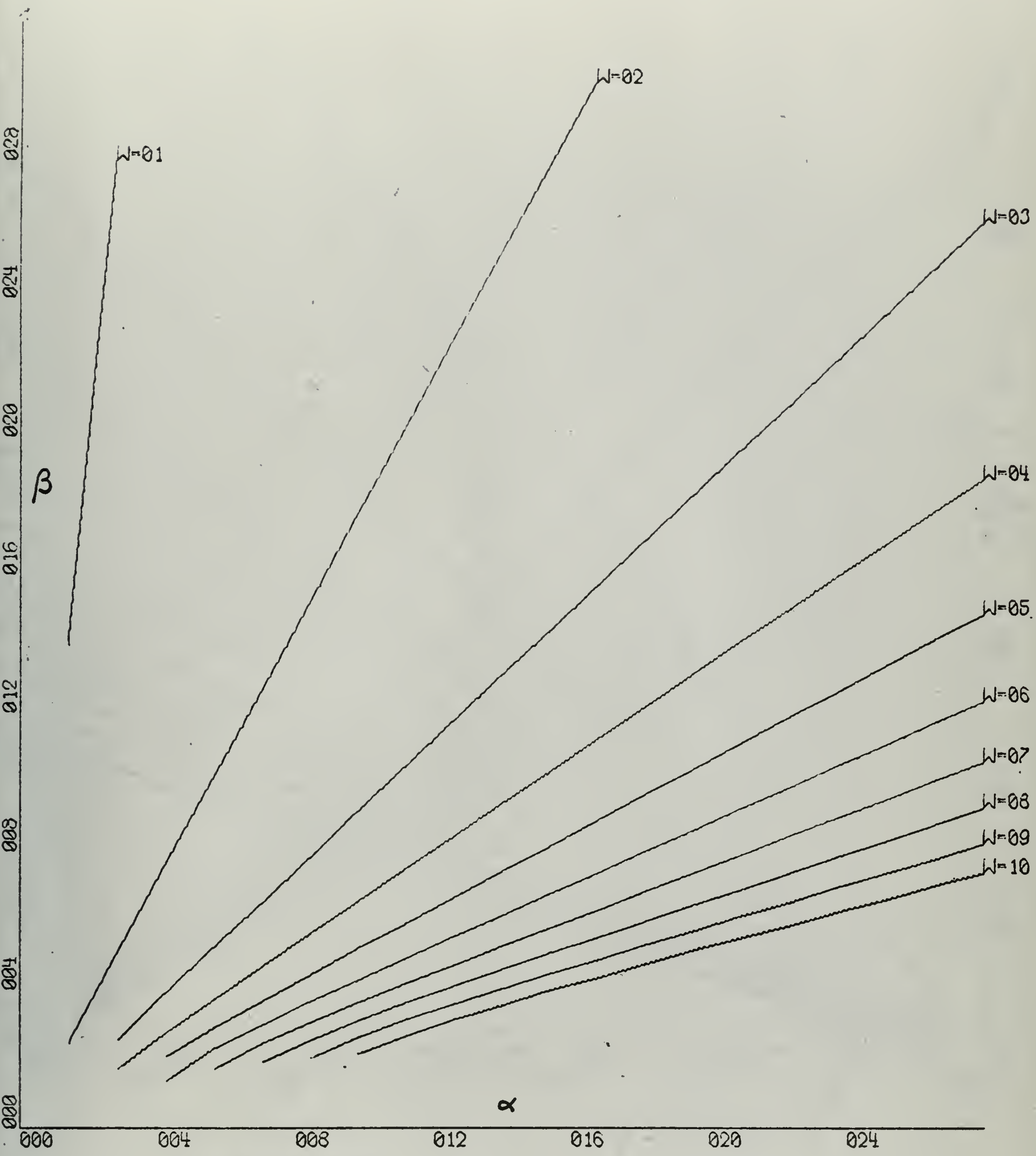


FIG 5-5

X-SCALE = 4.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. CONSTANT BANDWIDTH
10/S**2(S+1)

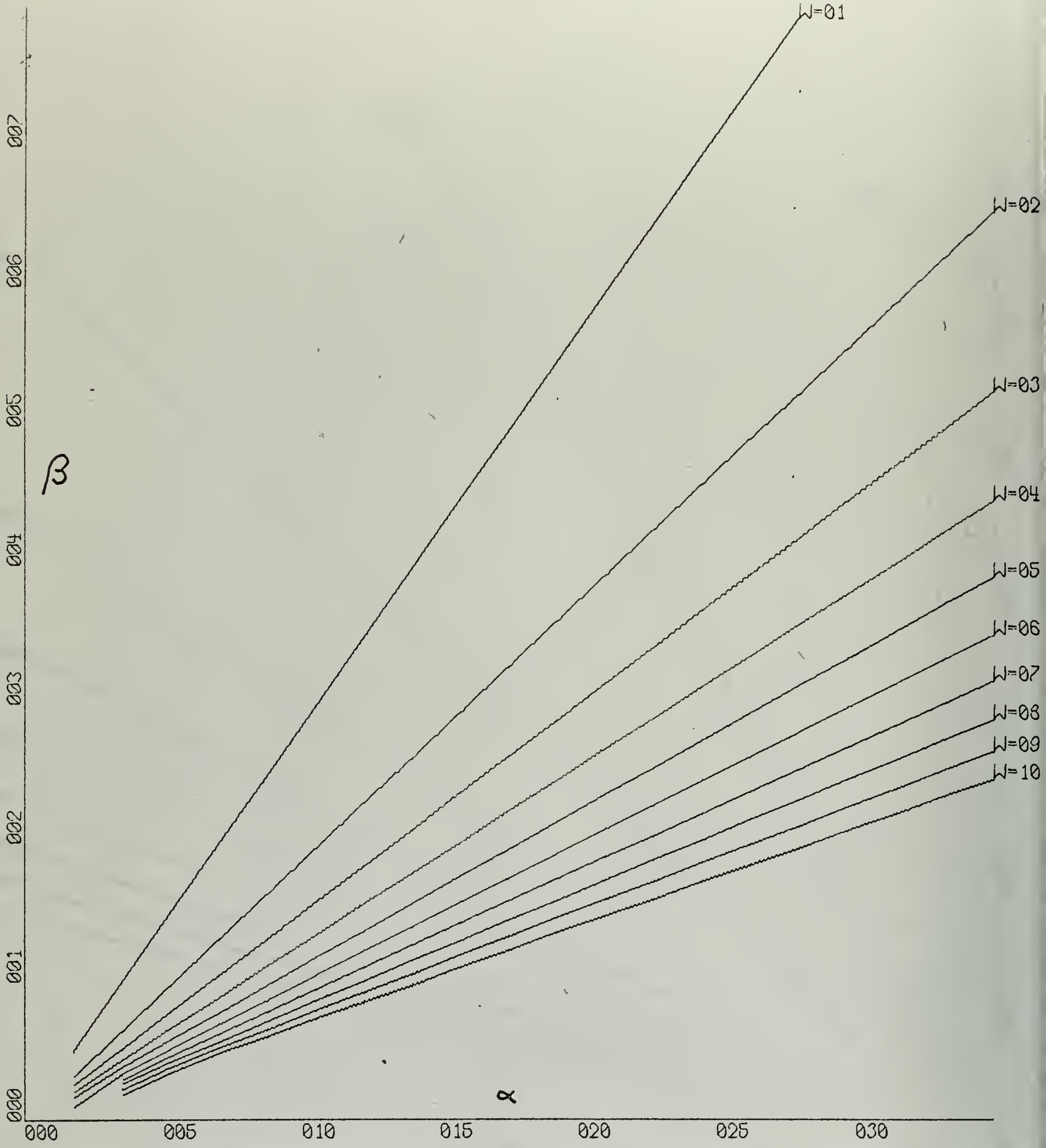


FIG 5 - 6

X-SCALE = 5.00E+01 UNITS/INCH.
Y-SCALE = 1.00E+01 UNITS/INCH.

PAINE, J.A., JR. 1/S**2(S+10)
CONSTANT BANDWIDTH

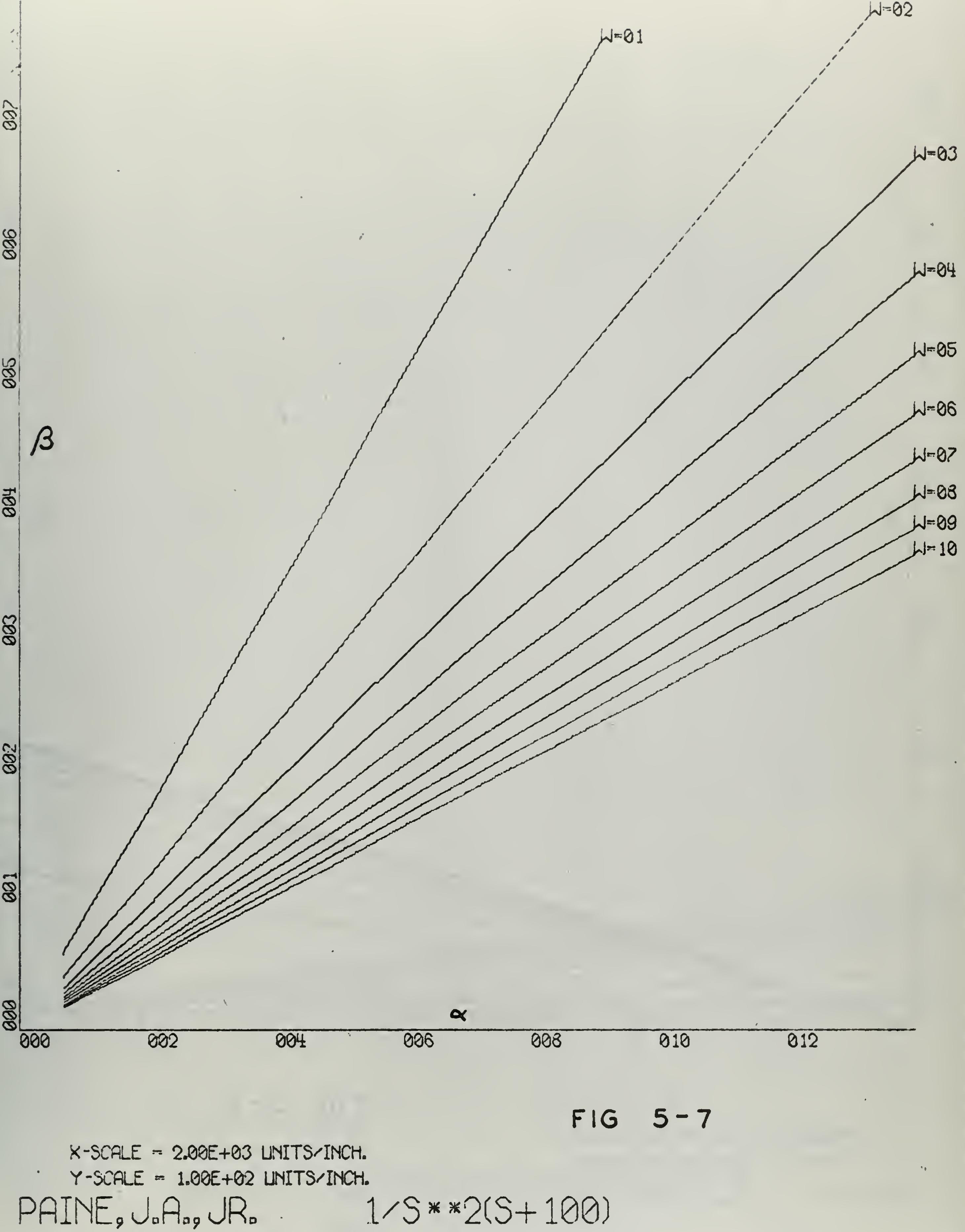


FIG 5-7

X-SCALE = 2.00E+03 UNITS/INCH.
Y-SCALE = 1.00E+02 UNITS/INCH.

PAINE, J.A., JR.   1/S**2(S+100)
CONSTANT BANDWIDTH

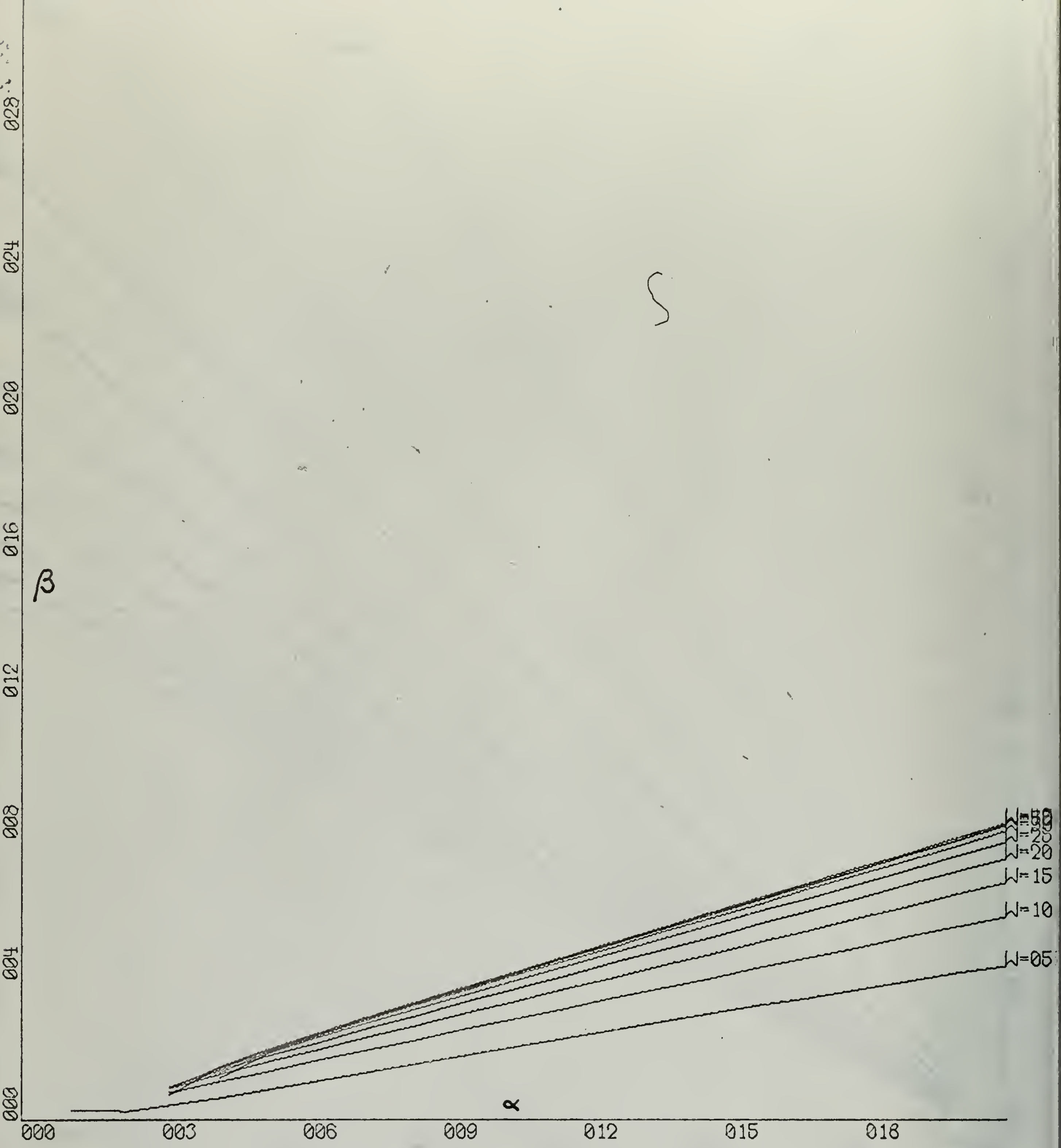


FIG 5-8

X-SCALE = 3.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. CONSTANT BANDWIDTH
25.0 / (S+5.0)(S+5.0)(S+10.0)

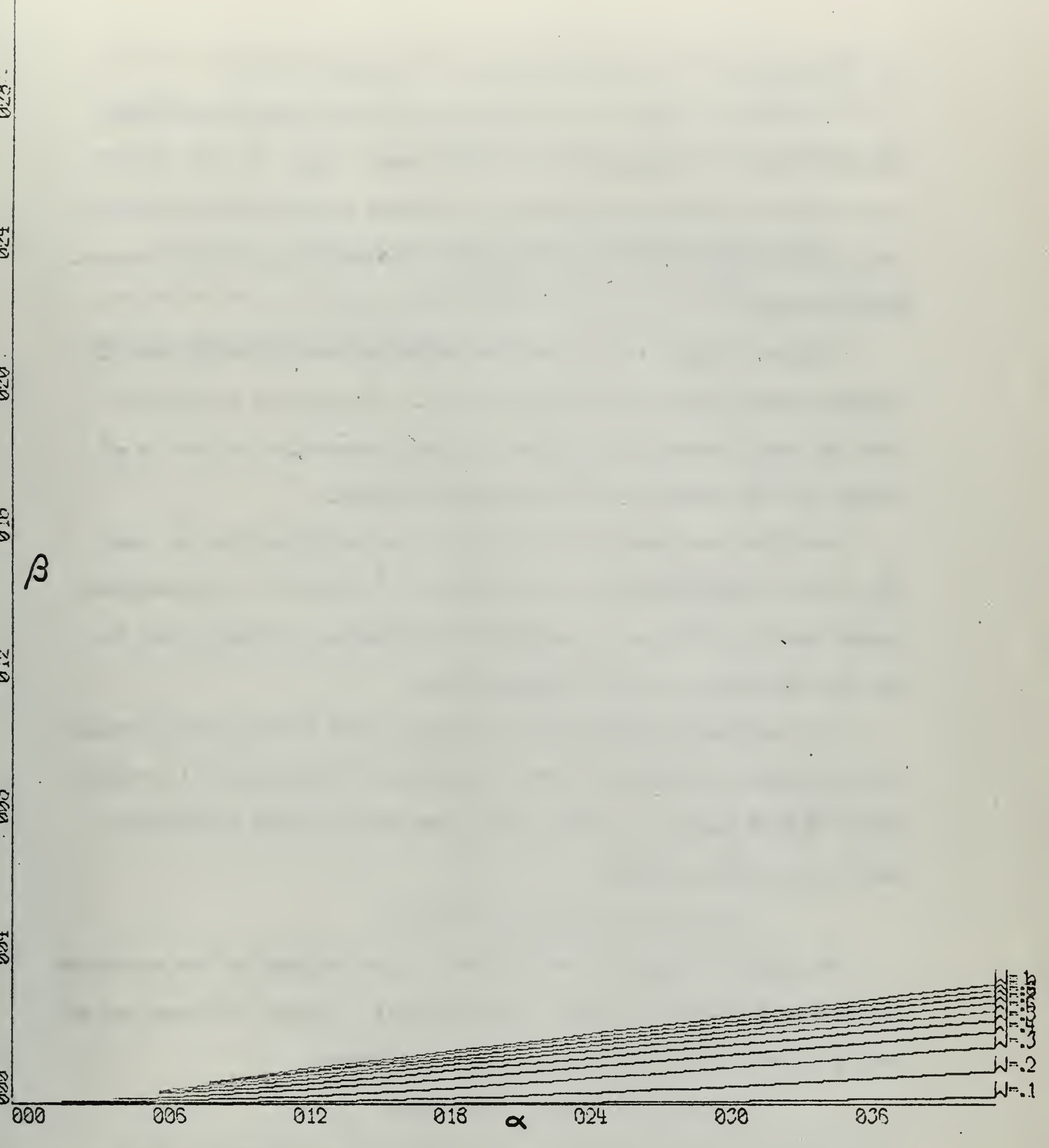


FIG 5-9

X-SCALE = 6.00E+00 UNITS/INCH.
Y-SCALE = 4.00E+00 UNITS/INCH.

PAINE, J.A., JR. CONSTANT BANDWIDTH
25.0 / (S+5.0)(S+5.0)(S+100.0)

6. Programing of the Compensation and Bandwidth Curves

Appendices III and IV are listings of the programs developed in the course of investigation of this work. Only PROGRAM PROJECT (Appendix III) will be explained, as PROGRAM PLOT (Appendix IV) is essentially PROGRAM ROOT LOCUS [6] with a minor subhead and internal modification.

PROGRAM PROJECT is designed to solve the α quadratic and β quartic established in Sections 4 and 5. The program is divided into two main sections, the first for the computation of the $\alpha-\beta$ points and the second for the bandwidth points.

The first section computes an 80 by 80 matrix of the α and β points corresponding to set values of ζ and ω. The computational part is followed by two distinct graphing sections, one for lag and the other for lead compensation.

The lag graphing section is set up so that during the plotting of the curves each value of α is tested to determine if its value is $10^{-7} \leq \alpha \leq 1.0001$. If no points are found within this range then a print out is made:

NO LAG COMPENSATION POSSIBLE

For the lead section graphs, α is again tested by the criterion $1.0001 \leq \alpha \leq$ (X-graph scale)(X graph width). Again if there are no values of α within this region the statement:

NO LEAD COMPENSATION POSSIBLE

is printed. In this case however a study of the printed values of α must be made to insure that the points are indeed non-existant or rather just lie outside the range of the graph.

The second main section of the program computes the value of

β for a given value of α as determined by the X graph scale. Here the plotting routine is set up so as to not plot zero points and to stop the curve when either the α or β value exceeds the range of the graph.

There are several go-no go options available within the program which are amply explained in the comment section preceeding the program listing in Appendix III.

7. Extensions to Higher Order Systems

Although the work presented to this point has been limited to third order systems and the program written for this specific case, investigation shows that generalized equations may be written which will allow the extension of the program to higher ordered systems. It can be shown that for a given n^{th} order system with no zeros to be compensated with two identical sections of cascade compensation, that the characteristic equation of the system may be generally written as:

$$s^{n+2} + 2ps^{n+1} + p^2 s^n + (z^2 s^2 + 2pzs + p^2)K +$$

$$2p \sum_{\substack{k=n\\ j=1}}^{\substack{j=n\\ k=1}} \left(\sum \prod_{j} r_i\right) s^k + p^2 \sum_{\substack{k=n-1\\ j=1}}^{\substack{j=n\\ k=0}} \left(\sum \prod_{j} r_i\right) s^k +$$

$$\sum_{\substack{k=n+1\\ j=1}}^{\substack{j=n\\ k=2}} \left(\sum \prod_{j} r_i\right) s^k = 0 \qquad (7\text{-}1)$$

where for n=4 the equation would be written:

$$s^6 + 2ps^5 + p^2 s^4 + (z^2 s^2 + 2pzs + p^2)K +$$

$$2p\left(\sum \prod_1 r_i s^4 + \sum \prod_2 r_i s^3 + \sum \prod_3 r_i s^2 + \sum \prod_4 r_i s\right) +$$

$$p^2\left(\sum \prod_1 r_i s^3 + \sum \prod_2 r_i s^2 + \sum \prod_3 r_i s + \sum \prod_4 r_i\right) +$$

$$\left(\sum \prod_1 r_i s^5 + \sum \prod_2 r_i s^4 + \sum \prod_3 r_i s^3 + \sum \prod_4 r_i s^2\right) = 0 \qquad (7\text{-}2)$$

It may be further shown that the parameters defined by Equations (4-4) through (4-13) may be written:

$$B_{21} = K\omega^2 U_1(\xi) \tag{7-3}$$

$$B_{22} = K\omega^2 U_2(\xi) \tag{7-4}$$

$$D_1 = -2K\omega U_o(\xi) \tag{7-5}$$

$$D_2 = -2K\omega U_1(\xi) \tag{7-6}$$

$$E_{11} = 2(-1)^{n+1}\omega^{n+1}U_n(\xi) + 2\sum_{\substack{k=n\\j=1}}^{\substack{j=n\\k=1}} \left[(-1)^k\omega^k U_{k-1}(\xi)\left(\sum\prod_j r_i\right)\right] \tag{7-7}$$

$$E_{12} = 2(-1)^{n+1}\omega^{n+1}U_{n+1}(\xi) + 2\sum_{\substack{k=n\\j=1}}^{\substack{j=n\\k=1}} \left[(-1)^k\omega^k U_k(\xi)\left(\sum\prod_j r_i\right)\right] \tag{7-8}$$

$$F_{21} = (-1)^n\omega^n U_{n-1}(\xi) + \sum_{\substack{k=n-1\\j=1}}^{\substack{j=n\\k=0}} \left[(-1)^k\omega^k U_{k-1}(\xi)\left(\sum\prod_j r_i\right)\right] + KU_{-1}(\xi) \tag{7-9}$$

$$F_{22} = (-1)^n\omega^n U_n(\xi) + \sum_{\substack{k=n-1\\j=1}}^{\substack{j=n\\k=0}} \left[(-1)^k\omega^k U_k(\xi)\left(\sum\prod_j r_i\right)\right] + KU_o(\xi) \tag{7-10}$$

$$G_1 = (-1)^{n+2}\omega^{n+2}U_{n+1}(\xi) + \sum_{\substack{k=n+1\\j=1}}^{\substack{j=n\\k=2}} \left[(-1)^k\omega^k U_{k-1}(\xi)\left(\sum\prod_j r_i\right)\right] \tag{7-11}$$

$$G_2 = (-1)^{n+2}\omega^{n+2}U_{n+2}(\xi) + \sum_{\substack{k=n+1\\j=1}}^{\substack{j=n\\k=2}} \left[(-1)^k\omega^k U_k(\xi)\left(\sum\prod_j r_i\right)\right] \tag{7-12}$$

These then are the recursive equations required for the complete generalization to a n^{th} order system. By employing the above equations and replacing in PROGRAM PROJECT (Appendix III) cards 100 through 150 and 300 through 540 with the appropriate programing, the program may be used for any given n^{th} order system.

In like manner by generally defining:

$$P(s) = \alpha^2 K s^2 + \alpha\beta K s + \beta^2 K \tag{7-13}$$

and:

$$Q(s) = s^{n+2} + 2\beta s^{n+1} + \beta^2 s^n + 2 \sum_{\substack{k=n\\ j=1}}^{\substack{j=n\\ k=1}} \left(\sum \prod_{j} r_i\right) s^k +$$

$$\beta^2 \sum_{\substack{k=n-1\\ j=1}}^{\substack{j=n\\ k=0}} \left(\sum \prod_{j} r_i\right) s^k + \sum_{\substack{k=n+1\\ j=1}}^{\substack{j=n\\ k=2}} \left(\sum \prod_{j} r_i\right) s^k \tag{7-14}$$

and using Equations (5-8) through (5-11) we may replace in the program cards 2860 and 2880 through 2920, thus adapting this part of the program to a general n^{th} order application.

8. Conclusions

Throughout this development of the Parameter Plane quadratic extension, the c_k's in the generalized coefficient form:

$$\sum_{k=0}^{n} (b_k\alpha^2 + c_k\alpha + d_k\alpha\beta + e_k\beta + f_k\beta^2 + g_k) = 0 \qquad (8\text{-}1)$$

have been identically zero. This at first appearance might seem to detract from the generalization. The inclusion of this parameter does not however introduce any great difficulty in the solution. The change in the development would be to the value of P_1 and P_2 which would become:

$$P_1 = C_1 + \beta D_1 \qquad (8\text{-}2)$$

$$P_2 = C_2 + \beta D_2 \qquad (8\text{-}3)$$

and the final solution for α which would change to:

$$\alpha = -\frac{P_1}{2B_{21}} \pm \left[\frac{P_1^2 - 2B_{21}Q_1}{4B_{21}^2}\right]^{\frac{1}{2}} \qquad (8\text{-}4)$$

For this case, new selection rules for acceptable values of α would be used, and would be much like those presented for β.

Though the extension of the Parameter Plane to include the $\alpha - \beta$ quadratic case makes this tool even more useful, further work is still to be done in this field. Not only must the equations for the solutions of the Parameter Plane curves for such cases as:

$$a_k = b_k\alpha^2\beta^2 + c_k\alpha^2\beta + d_k\alpha\beta^2 + e_k\alpha^2 + f_k\beta^2 + g_k\alpha\beta + h_k\alpha + q_k\beta + t_k \qquad [5]\ (8\text{-}5)$$

and higher ordered combinations of the parameters be developed, but more efficient programing techniques must be developed. In the use of PROGRAM PROJECT, for instance, as the location of the system poles on the σ axis of the S-plane move to the left, the computational time becomes excessive due to present programing technique and computer speed.

Another major problem in further extensions of these techniques, and indeed even other applications of the curves from the preceeding development, will be interpretation. In this case, the initial substitution of variables immediately allowed interpretation of the curves sight unseen. Here then, will be most likely the one single drawback to further extension, for as the parameters α and β are used as representations of other variables in control systems, each application will have its own unique interpretation.

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APPENDIX I

THE ROSS-WARREN METHOD OF COMPENSATION

The Ross-Warren [2] Method is an algebraic way of determining the location of compensator poles and zeros for single or multiple sections of cascade lead or lag compensation. The relationships of the method are based on relationships between the root locus gain and the error coefficients, where the uncompensated root locus gain is:

$$\left|K(r)\right| = \frac{\left|r\right|^{n} \prod_{i=1}^{n} \left|r + p_i\right|}{\prod_{i=1}^{m} \left|r + z_i\right|} \tag{I-1}$$

and the compensated root locus gain at the point r is:

$$K_c(r) = K(r)\left(\frac{p_d}{z_d}\right)^{k} \tag{I-2}$$

where k is the number of identical sections introduced by the compensator.

The other part of the equality for the derivation is determined by the fact that the uncompensated error coefficient is defined by:

$$K_x(r) = K(r)\,\frac{\prod_{i=1}^{m} z_i}{\prod_{i=1}^{n} p_i} \tag{I-3}$$

and the compensated error coefficient is:

$$K_{cx}(r) = K_x(r)\cdot\frac{p_d}{z_d}\cdot\frac{z_c}{p_c} \tag{I-4}$$

These relationships are expressed geometrically on the S-plane in Figure I-1 and are used to derive the relationships:

S-PLANE RELATIONSHIPS TO DERIVE THE THE ROSS-WARREN EQUATIONS

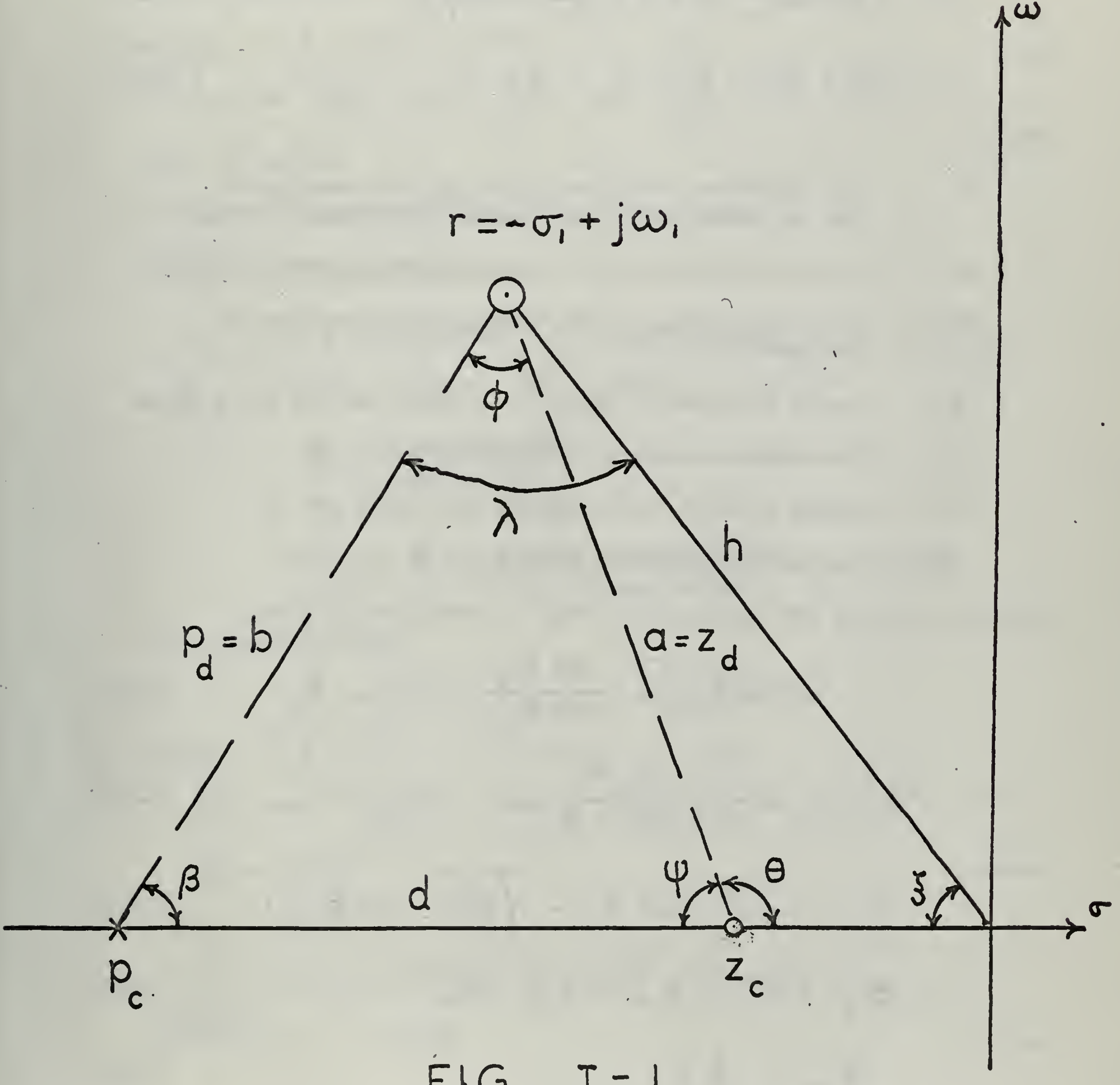

FIG I-1

$$z_c = \frac{hK \sin \lambda}{K(r) \sin \theta} \tag{I-5}$$

$$p_c = \frac{h \sin \lambda}{\sin (\theta - \phi)} \tag{I-6}$$

where:

$$\lambda = \cot^{-1} \left(\cot \phi - \frac{K}{K(r)} \csc \phi \right) \tag{I-7}$$

and:

$$\theta = \pi + \phi - \xi - \lambda \tag{I-8}$$

Here:

h - the distance from the origin to the root point and is specified by the choice of zeta and omega

K - root locus gain of the uncompensated system

$K(r)$ - root locus gain of the uncompensated system at the point r

ϕ - angle required to make r a point on the root locus

λ - an angle calculable from K, $K(r)$ and ϕ

ξ - angle between the negative real axis and h

θ - an angle calculable from ϕ, ξ and λ

These equations are generalized for n sections so that:

$$z_c = h\sqrt[n]{\frac{K}{K(r)}} \; \frac{\sin \lambda_n}{\sin \theta_n} \tag{I-9}$$

$$p_c = \frac{h \quad \sin \lambda_n}{\sin (\theta_n - \phi_n)} \tag{I-10}$$

where:

$$\lambda_n = \cot^{-1} \left(\cot \phi_n - \sqrt[n]{\frac{K}{K(r)}} \csc \phi_n\right) \tag{I-11}$$

$$\theta_n = \pi + \phi_n - \xi - \lambda_n \tag{I-12}$$

and:

$$\phi_n = \phi / n \tag{I-13}$$

APPENDIX II

DETERMINATION OF BETA QUARTIC SOLUTION

Here, three examples of the approximately fifty calculations that were made in the investigation of the proper solutions of α and β, will be presented. These calculations are started by employing Equations (I-9) through (I-13) with n equal to two.

Case 1. $\zeta = 90^\circ$ $K = 30.0$ $h = 4.0$

$$\phi = \tan^{-1}\frac{4.0}{0.0} + \tan^{-1}\frac{4.0}{1.0} + \tan^{-1}\frac{4.0}{5.0} - 180^\circ = 24^\circ 38'$$

$$\phi_2 = 12^\circ 19'$$

$$K(r) = (4.0)(17.0)^{\frac{1}{2}}(41.0)^{\frac{1}{2}} = 105.60304$$

$$\sqrt{K/K(r)} = .532994$$

$$\lambda_2 = \cot^{-1}(\cot 12^\circ 19' - (.532994)\csc 12^\circ 19') = 25^\circ 40'$$

$$\theta_2 = 180^\circ + 12^\circ 19' - 90^\circ - 25^\circ 40' = 76^\circ 39'$$

$$p_c = \frac{4.0 \sin 25^\circ 40'}{\sin 64^\circ 20'} = 1.9225$$

$$z_c = (4.0)(.532994)\frac{\sin 25^\circ 40'}{\sin 76^\circ 39'} = .94906$$

this implies that:

$$\beta = 1.9225 \qquad \alpha = 2.02568$$

Case 2. $\zeta = 60^\circ$ $K = 30.0$ $h = 5.0$

$$\phi = \tan^{-1}\left(\frac{4.3302}{2.5}\right) + \tan\left(-\frac{4.3302}{1.5}\right) + \tan\left(-\frac{4.3302}{2.5}\right) - 180^\circ$$

$$\phi = 109^\circ 16'$$

$$\phi_2 = 54^\circ 33'$$

$$K(r) = (5.0)(21.000199)^{\frac{1}{2}}(25.00019)^{\frac{1}{2}} = 114.8764$$

$$\sqrt{K/K(r)} = .511029$$

$$\lambda_2 = \cot^{-1}\left(\cot 54^\circ 33' - (.511029)\csc 54^\circ 33'\right) = 85^\circ 09.5'$$

$$\theta_2 = 180^\circ + 54^\circ 33' - 60^\circ - 85^\circ 09.5' = 89^\circ 32.5'$$

$$p_c = \frac{(5.0)\ \sin 85^\circ 09.5'}{\sin 34^\circ 50.5'} = 8.72057$$

$$z_c = (5.0)(.511029)\ \frac{\sin 85^\circ 09.5'}{\sin 89^\circ 23.5'} = 2.54616$$

this implies that:

$$\beta = 8.72057 \qquad \alpha = 3.42498$$

Case 3. $\xi = 90^\circ$ $K = 30.0$ $h = 5.0$

$$\phi = \tan^{-1}\left(\frac{5.0}{0.0}\right) + \tan^{-1}\left(\frac{5.0}{1.0}\right) + \tan^{-1}\left(\frac{5.0}{5.0}\right) - 180^\circ = 33^\circ 48'$$

$$\phi_2 = 16^\circ 54'$$

$$K(r) = (5.0)(26.0)^{\frac{1}{2}}(50.0)^{\frac{1}{2}} = 180.277586$$

$$\sqrt{K/K(r)} = .407934$$

$$\lambda_2 = \cot^{-1}\left(\cot 16^\circ 54' - (.407934)\csc 16^\circ 54'\right) = 27^\circ 59'$$

$$\theta_2 = 180^\circ + 16^\circ 54' - 90^\circ - 27^\circ 59' = 78^\circ 55'$$

$$p_c = \frac{(5.0)\ \sin 27^\circ 59'}{\sin 62^\circ 01'} = 2.60005$$

$$z_c = (5.0)(.407934)\frac{\sin 27^\circ 59'}{\sin 78^\circ 55'} = .95444$$

this implies that:

$$\beta = 2.60005 \qquad \alpha = 2.72416$$

Now taking the values of β just found we compare them to the roots of the β quartic given in Figures II-1b, II-2b and II-3b respectively. We see that the proper answer for β, in all cases, is indeed the largest positive and real root of the quartic. From

Figures II-4 and II-5 we now obtain the values necessary to solve for the value of α . These steps for the three cases are shown in Figure II-6, and again we note that only when applying the above selected values of β do we obtain the correct value of α .

FIG II-1A

```
ZETA = .00          OMEGA =   3.5

THE COEFFICIENTS OF THE GIVEN POLYNOMIAL ARE


REAL PART
       1.0000000       .8178700     24.0270000    -54.5470000     16.4780000
IMAGINARY PART
        .0000000       .0000000       .0000000       .0000000       .0000000


                                         TABLE OF ROOTS

  ROOT          REAL  PART          IMAG  PART          REAL  PART          IMAG  PART
 NUMBER          OF  ROOT            OF  ROOT            OF  P(Z)            OF  P(Z)
    1                .360270            .000000            -.000000            .000000
    2               1.586139            .000000            -.000000            .000000
    3              -1.382140          -5.188995             .000000            .000000
    4              -1.382140           5.188995             .000000           -.000000
```

FIG II-1B

```
ZETA = .00          OMEGA =   4.0

THE COEFFICIENTS OF THE GIVEN POLYNOMIAL ARE


REAL PART
       1.0000000      2.5714000     34.8570000   -109.7100000     50.2860000
IMAGINARY PART
        .0000000       .0000000       .0000000       .0000000       .0000000


                                         TABLE OF ROOTS

  ROOT          REAL  PART          IMAG  PART          REAL  PART          IMAG  PART
 NUMBER          OF  ROOT            OF  ROOT            OF  P(Z)            OF  P(Z)
    1                .564892            .000000             .000000            .000000
    2               1.920874            .000000            -.000000            .000000
    3              -2.528583          -6.320531             .000000            .000000
    4              -2.528583           6.320531             .000000           -.000000
```

```
ZETA = .00          OMEGA =    4.5
                                                                    FIG  II-2A
THE COEFFICIENTS OF THE GIVEN POLYNOMIAL ARE


REAL PART
      1.0000000        4.8420000       45.2390000     -193.7100000     123.0900000
IMAGINARY PART
       .0000000         .0000000         .0000000         .0000000        .0000000


                                       TABLE OF ROOTS

  ROOT           REAL  PART          IMAG  PART          REAL  PART          IMAG  PART
 NUMBER           OF  ROOT            OF  ROOT            OF  P(Z)            OF  P(Z)
   1                 .799670            .000000             .000000             .000000
   2                2.272470            .000000            -.000000             .000000
   3               -3.957070          -7.216420             .000000             .000000
   4               -3.957070           7.216420             .000000            -.000000


ZETA = .00          OMEGA =    5.0
                                                                    FIG  II-2B
THE COEFFICIENTS OF THE GIVEN POLYNOMIAL ARE


REAL PART
      1.0000000        7.3770000       53.2790000     -307.3800000     256.1500000
IMAGINARY PART
       .0000000         .0000000         .0000000         .0000000        .0000000


                                       TABLE OF ROOTS

  ROOT           REAL  PART          IMAG  PART          REAL  PART          IMAG  PART
 NUMBER           OF  ROOT            OF  ROOT            OF  P(Z)            OF  P(Z)
   1                1.061458            .000000             .000000             .000000
   2                2.638460            .000000            -.000000             .000000
   3               -5.538459          -7.796637             .000000            -.000000
   4               -5.538459           7.796637             .000000             .000000
```

FIG II-3A

```
ZETA = .5     OMEGA =      4.5

THE COEFFICIENTS OF THE GIVEN POLYNOMIAL ARE


REAL PART
      1.0000000      -5.4234000      -15.7540000    -112.1700000     411.2800000
IMAGINARY PART
       .0000000        .0000000        .0000000        .0000000        .0000000


                                  TABLE OF ROOTS

  ROOT          REAL   PART          IMAG   PART          REAL   PART          IMAG   PART
 NUMBER          OF   ROOT            OF   ROOT            OF   P(Z)            OF   P(Z)
    1               2.441988              .000000              .000000              .000000
    2              -2.633771             3.671566              .000004              .000001
    3              -2.633771            -3.671566              .000000              .000004
    4               8.248955             -.000000             -.000002             -.000003
```

FIG II-3B

```
ZETA = .5     OMEGA =      5.0

THE COEFFICIENTS OF THE GIVEN POLYNOMIAL ARE


REAL PART
      1.0000000      -5.2147000      -27.6070000     -95.8590000     599.1200000
IMAGINARY PART
       .0000000        .0000000        .0000000        .0000000        .0000000


                                  TABLE OF ROOTS

  ROOT          REAL   PART          IMAG   PART          REAL   PART          IMAG   PART
 NUMBER          OF   ROOT            OF   ROOT            OF   P(Z)            OF   P(Z)
    1               3.011147              .000000              .000001              .000000
    2              -3.264834             3.481907              .000001             -.000000
    3              -3.264834            -3.481907              .000001              .000000
    4               8.733221              .000000              .000001              .000000
```

```
VALUES OF THE BETA COEFFICIENTS FOR ZETA =      .00000

  OMEGA        BETA**4        BETA**3        BETA**2          BETA          CONSTANT


   .50000     .19206E+06    -.15630E+05     .25633E+04    -.10020E+03    -.19830E+02
  1.00000      25776E+07    -.77760E+06     .54720E+06    -.86400E+05    -.14400E+05
  1.50000     .89438E+07    -.52437E+07     .94830E+07    -.34253E+07    -.39248E+06
  2.00000     .10310E+08    -.82944E+07     .33638E+08    -.22118E+08    -.92160E+06
  2.50000    -.31984E+08     .24719E+08    -.24290E+09     .25749E+09    -.13411E+08
  3.00000    -.22045E+09     .62986E+08    -.32123E+10     .51018E+10    -.85031E+09
  3.50000    -.79196E+09    -.64771E+09    -.19028E+11     .43199E+11    -.13050E+11
  4.00000    - 22708E+10    -.58393E+10    -.79154E+11     .24914E+12    -.11419E+12
  4.50000    -.57903E+10    -.28036E+11    -.26194E+12     .11216E+13    -.71270E+12
  5.00000    -.13725E+11    -.10125E+12    -.73125E+12     .42188E+13    -.35156E+13

THE NORMALIZED COEFFICIENTS ARE

   .50000     .10000E+01    -.81384E-01     .13347E-01    -.52169E-03    -.10325E-03
  1.00000     .10000E+01    -.30168E+00     .21229E+00    -.33520E-01    -.55866E-02
  1.50000     .10000E+01    -.58629E+00     .10603E+01    -.38298E+00    -.43883E-01
  2.00000     .10000E+01    -.80447E+00     .32626E+01    -.21453E+01    -.89385E-01
  2.50000     .10000E+01    -.77286E+00     .75945E+01    -.80507E+01     .41931E+00
  3.00000     .10000E+01    -.28571E+00     .14571E+02    -.23143E+02     .38571E+01
  3.50000     .10000E+01     .81787E+00     .24027E+02    -.54547E+02     .16478E+02
  4.00000     .10000E+01     .25714E+01     .34857E+02    -.10971E+03     .50286E+02
  4.50000     .10000E+01     .48420E+01     .45239E+02    -.19371E+03     .12309E+03
  5.00000     .10000E+01     .73770E+01     .53279E+02    -.30738E+03     .25615E+03

THE RUCURRING VALUES ARE       W          D1           E11         F21          G1
                           .500E+00    .000E+00     .238E+01   -.285E+02   -.375E+00
                           .100E+01    .000E+00     .800E+01   -.240E+02   -.600E+01
                           .150E+01    .000E+00     .124E+02   -.165E+02   -.304E+02
                           .200E+01    .000E+00     .800E+01   -.600E+01   -.960E+02
                           .250E+01    .000E+00    -.156E+02    .750E+01   -.234E+03
                           .300E+01    .000E+00    -.720E+02    .240E+02   -.486E+03
                           .350E+01    .000E+00    -.178E+03    .435E+02   -.900E+03
                           .400E+01    .000E+00    -.352E+03    .660E+02   -.154E+04
                           .450E+01    .000E+00    -.618E+03    .915E+02   -.246E+04
                           .500E+01    .000E+00    -.100E+04    .120E+03   -.375E+04
```

FIG II-4

```
    VALUES OF THE BETA COEFFICIENTS FOR ZETA =     .50000

  OMEGA        BETA**4        BETA**3        BETA**2          BETA         CONSTANT


   .50000     .15029E+06    -.11023E+05    .31245E+04    -.38672E+02    -.35156E+01
  1.00000     .20916E+07    -.25200E+06    .38880E+06     .18000E+05    -.90000E+03
  1.50000     .78162E+07     .26795E+07    .40539E+07     .37367E+07    -.83038E+06
  2.00000     .12902E+08     .45158E+08    .00000E+00     .10322E+09    -.45158E+08
  2.50000    -.42984E+07     .27260E+09   -.13905E+09     .12016E+10    -.85831E+09
  3.00000    -.10235E+09     .10550E+10   -.70859E+09     .82905E+10    -.89814E+10
  3.50000    -.42426E+09     .31465E+10   -.98367E+09     .39723E+11    -.63557E+11
  4.00000    -.13170E+10     .80806E+10    .69009E+10     .14345E+12    -.34068E+12
  4.50000    -.36064E+10     .19559E+11    .56814E+11     .40452E+12    -.14832E+13
  5.00000    -.91688E+10     .47813E+11    .25312E+12     .87891E+12    -.54932E+13

THE NORMALIZED COEFFICIENTS ARE


   .50000     .10000E+01    -.73345E-01    .20790E-01    -.25731E-03    -.23392E-04
  1.00000     .10000E+01    -.12048E+00    .18589E+00     .86059E-02    -.43029E-03
  1.50000     .10000E+01     .34281E+00    .51865E+00     .47807E+00    -.10624E+00
  2.00000     .10000E+01     .35000E+01    .00000E+00     .80000E+01    -.35000E+01
  2.50000     .10000E+01    -.63419E+02    .32348E+02    -.27955E+03     .19968E+03
  3.00000     .10000E+01    -.10308E+02    .69231E+01    -.81000E+02     .87750E+02
  3.50000     .10000E+01    -.74165E+01    .23185E+01    -.93628E+02     .14980E+03
  4.00000     .10000E+01    -.61358E+01   -.52400E+01    -.10892E+03     .25869E+03
  4.50000     .10000E+01    -.54234E+01   -.15754E+02    -.11217E+03     .41128E+03
  5.00000     .10000E+01    -.52147E+01   -.27607E+02    -.95859E+02     .59912E+03

THE RUCURRING VALUES ARE          W          D1          E11         F21         G1
                              .500E+00   .000E+00    .100E+01   -.286E+02   -.594E+00
                              .100E+01   .000E+00   -.200E+01   -.250E+02   -.400E+01
                              .150E+01   .000E+00   -.180E+02   -.199E+02   -.928E+01
                              .200E+01   .000E+00   -.560E+02   -.140E+02   -.800E+01
                              .250E+01   .000E+00   -.125E+03   -.813E+01    .195E+02
                              .300E+01   .000E+00   -.234E+03   -.300E+01    .108E+03
                              .350E+01   .000E+00   -.392E+03    .625E+00    .311E+03
                              .400E+01   .000E+00   -.608E+03    .200E+01    .704E+03
                              .450E+01   .000E+00   -.891E+03    .375E+00    .139E+04
                              .500E+01   .000E+00   -.125E+04   -.500E+01    .250E+04
```

FIG II- 5

ξ	ω	β	β^2	βE_{11}	$\beta^2 F_{21}$	Q_1/B_{21}	α	Z_c
0.0	4.0	0.564892	0.319103	-198.841984	21.060798	-3.57871	1.891	0.299
0.0	4.0	1.920874	3.6895569	-676.147648	243.523955	-4.10963	2.028	0.947
0.0	5.0	1.061458	1.1266931	-1061.4580	135.203232	-6.2350	2.497	0.425
0.0	5.0	2.638460	6.9614712	-2638.460	835.376544	-7.40411	2.721	0.970
0.5	5.0	3.011147	9.067006	-3763.93375	-45.33503	-1.74519	1.3212	2.279
0.5	5.0	8.733221	76.269149	-10916.52625	-381.345745	-11.73049	3.425	2.550

FIG. II-6

APPENDIX III

PROGRAM PROJECT

```
..JOB0473F, PAINE, J.A.,JR. - THESIS
      PROGRAM PROJECT                                                          000000
C  THIS PROGRAM COMPUTES THE VALUES OF BETA(POLE LOCATION) AND ALFA(POLE
C    -ZERO RATIO) BASED ON PARAMETER PLANE TECHNIQUES.  THE COMPUTED
C     ALFA AND BETA VALUES ARE THOSE REQUIRED TO PLACE THE ROOTS OF ANY
C     THIRD ORDER SYSTEM, TYPE 0,1,2 OR 3, AT A DESIRED ZETA AND OMEGA
C    LOCATION WHILE MAINTAINING A CONSTANT VELOCITY COEFFICIENT.  AFTER
C     COMPUTING THE VALUES IT WILL PLOT THE PARAMETER PLANE CONSTANT ZETA
C    CURVES FROM 0.0 TO 0.9 AND THE CONSTANT OMEGA CURVES FOR EVEY ONE
C     TENTH OF THE VALUE OF THE MAXIMUM VALUE OF OMEGA USED.  A 9 BY 15
C     INCH GRAPH IS OUTPUT BY THE ROUTINE.  THIS IS DONE
C     ON TWO SEPARATE GRAPHS, ONE FOR POSSIBLE LAG COMPENSATION AND ONE
C    FOR LEAD COMPENSATION.  THE PROGRAM THEN HAS THE ADDITIONAL FEATURE
C     OF COMPUTING AND PLOTTING THE CONSTANT BANDWIDTH CURVES.
C  THE FOLLOWING FEATURES ARE AVAILABLE WITH THE PROPER USE OF THE DATA
C     CARDS.
C    1. THE ALFA-BETA COMPUTATIONS MAY OR MAY NOT BE DONE.
C    2. LAG COMPENSATION MAY OR MAY NOT BE PLOTTED.
C    3. LEAD COMPENSATION MAY OR MAY NOT BE PLOTTED.
C    4. BANDWIDTH COMPUTATIONS MAY OR MAY NOT BE COMPLETED.
C    5. BANDWIDTH CURVES MAY OR MAY NOT BE PLOTTED. (AVAILABLE ONLY IF
C         THE BANDWIDTH COMPUTATIONS HAVE BEEN MADE)
C  THE FOLLOWING DATA CARDS ARE REQUIRED.
C***CARD ONE - A,B,C,G - TEN COLUMNS PER NUMBER IN FLOATING POINT.
C      THESE ARE THE LOCATIONS OF THE UNCOMPENSATED POLES AND THE
C    UNCOMPENSATED ROOT LOCUS GAIN.
C***CARD TWO - WFIN - TEN COLUMNS IN FLOATING POINT.
C    THIS IS THE MAXIMUM VALUE OF OMEGA TO BE USED.
C***CARD THREE - IABCMP - COLUMN ONE IN FIXED POINT
C     0 - THE ALFA-BETA COMPUTATIONS WILL BE DONE
C     1 - THE ALFA-BETA COMPUTATIONS WILL NOT BE DONE
C                           *****
C   IF IABCMP=1 CARDS FOUR THROUGH THIRTEEN ARE OMITTED
C                           *****
C***CARD FOUR - ILGPLT - COLUMN ONE IN FIXED POINT
C     0 - THE LAG ZONE CURVES WILL BE PLOTTED
```

```
C      1 - THE LAG ZONE CURVES WILL NOT BE PLOTTED
C                      ******
C    IF ILGPLT=1 THE NEXT FOUR CARDS ARE OMITTED
C                      ******
C***CARD FIVE -IT(1)-IT(6) - COLUMNS 1-48 IN ALFANUMERIC CHARACTERS
C     THIS IS THE FIRST LINE OF THE LAG GRAPH TITLE
C***CARD SIX - IT(7)-IT(12) - COLUMNS 1-48 IN ALFANUMERIC CHARACTERS
C     THIS IS THE SECOND LINE OF THE LAG GRAPH TITLE.
C***CARD SEVEN - LBL(11)-LBL(20) - FOUR COLUMNS PER LABEL (TEN LABELS IN
C      CONSECUTIVE COLUMNS) IN ALFANUMERIC CHARACTERS.
C     THESE ARE THE LABELS TO BE PUT ON THE CONSTANT OMEGA CURVES.   TO
C      DETERMINE WHICH VALUES WILL BE PLOTTED, DIVIDE WFIN BY 10 .  THIS
C      VALUE AND INTEGER MULTIPLES OF IT TO 10 WILL BE PLOTTED.
C***CARD EIGHT - XLGZ,YLGZ - TEN COLUMNS PER NUMBER IN EXPONENTIAL OR
C      FLOATING POINT.
C     THESE ARE THE X AND Y SCALES FOR THE LAG GRAPH.  ONLY ONE SIGNI-
C     FICANT NUMBER IS TO BE USED.
C***CARD NINE - ILDPLT - COLUMN ONE IN FIXED POINT
C      0 - THE LEAD CURVES WILL BE PLOTTED
C      1 - THE LEAD CURVES WILL NOT BE PLOTTED
C                      ******
C    IF ILDPLT=1 THE NEXT FOUR CARDS ARE OMITTED
C                      ******
C***CARD TEN - THE SAME AS CARD FIVE EXCEPT FOR THE LEAD GRAPH
C***CARD ELEVEN - THE SAME AS CARD SIX EXCEPT FOR THE LEAD GRAPH
C***CARD TWELVE - THE SAME AS CARD EIGHT EXCEPT FOR THE LEAD GRAPH
C***CARD THIRTEEN - A***DUPLICATE*** OF CARD SEVEN
C***CARD FOURTEEN - IBWCMP - COLUMN ONE FIXED POINT
C      0 - BANDWIDTH COMPUTATIONS AND GRAPHING WILL NOT BE DONE.
C      1 - BANDWIDTH COMPUTATIONS WILL BE DONE
C                      ******
C    IF IBWCMP=0 THE REMAINING CARDS ARE OMITTED
C                      ******
C***CARD FIFTEEN - BWX,BWY - THE SAME AS CARD EIGHT EXCEPT FOR THE
C     BANDWIDTH CURVES.
C      BWY IS ALSO USED TO DETERMINE WHICH VALUES OF ALFA WILL BE USED IN
```

```
C     THE BANDWIDTH COMPUTATIONS.
C***CARD SIXTEEN - WEND - TEN COLUMNS IN FLOATING POINT
C     THIS IS THE MAXIMUM VALUE OF OMEGA FOR WHICH THE BANDWIDTH
C     COMPUTATIONS WILL BE DONE
C***CARD SEVENTEEN - IBWPLT - COLUMN ONE IN FIXED POINT
C     0 - THE BANDWIDTH CURVES WILL BE PLOTTED
C     1 - THE BANDWIDTH CURVES WILL NOT BE PLOTTED
C                 ******
C   IF IBWPLT=1 THE REMAINING CARDS ARE OMITTED
C                 ******
C***CARD EIGHTEEN - THE SAME AS CARD FIVE EXCEPT FOR THE BANDWIDTH CURVES
C***CARD NINETEEN - THE SAME AS CARD SIX EXCEPT FOR THE BANDWIDTH CURVES
C***CARD TWENTY - BANDWIDTH CURVE LABELS
C     TO DETERMINE WHICH CURVES WILL BE PLOTTED, DIVIDE  WEND  BY 10  .
C     THE PROGRAM PLOTS THIS CURVE AND INTEGER MULTIPLES OF IT UP TO 10.
C
C
C   IT IS RECOMMENDED THAT FOR THE INITIAL RUN THE FOLLOWING DATA CARDS
C     BE USED.
C        CARDS 1,2,3(IABCMP=0),4(ILGPLT=1),9(ILDPLT=1),14(IBWCMP=0)
C
C   THESE DATA CARDS WILL ALLOW ONLY THE ALFA-BETA COMPUTATIONS TO BE
C     COMPLETED.  A PRINT OUT OF THE VALUES WILL BE OUTPUT WHICH WILL ALLOW
C     YOU TO CHOOSE THE PROPER CURVES AND SCALES.  CAREFUL SELECTION
C     OF CURVE SCALES IS IMPORTANT, FOR THE PROGRAM WILL NOT ALLOW POINTS
C     OUTSIDE THE AXIX LIMITS TO BE PLOTTED.
C
      DIMENSION AFIN(80,80),BFIN(80,80),XAZ(80),YBZ(80),XAW(80),         000010
     1 YBW(80),IT(12),LBL(20),BCOFI(5),ROOTR(4),ROOTI(4),ACOFI(3),       000020
     2 U(10),AROOTI(4),ACOFR(3),BCOFR(5),WLAB(80),ZLAB(80),AROOTR(4)     000030
      COMMON BCOFR,BCOFI,ROOTR,ROOTI,BFINAL,IFLAG,AFIN,BFIN              000040
 9999 PRINT 140                                                          000050
  140 FORMAT (1H1)                                                       000060
      DO 60 JK=1,6400                                                    000070
      AFIN(JK) =0.0                                                      000080
   60 BFIN(JK) = 0.0                                                     000090
```

```
      READ 1,A,B,C,G
    1 FORMAT(4F10.0)
      PROD = A*B*C
      SUM = A+B+C
      SMPRD = A*B + A*C + B*C
      PRDGN = PROD + G
      ZETA = 0.0
      READ 2,WFIN
    2 FORMAT (F10.0)
      READ 9, IABCMP
    9 FORMAT (I1)
      IF(IABCMP-1)23,24,24
   23 STP = WFIN/80.
      DO 12 L = 1,80
      LJ = 80*(L-1)
      W = STP
   30 U(1)=-1.
      U(2)=0.
      U(3)=1.
      DO 10 N=2,6
   10 U(N+2)=2.*ZETA*U(N+1)-U(N)
      DO 11 J=1,80
      LJ = LJ + 1
      W2=W*W
      W3=W2*W
      W4=W2*W2
      W5=W2*W3
      CONN = G*W2
      CON = -2.*G*W
      CON1 = 2.*W4
      CON2 = -2.*SUM*W3
      CON3 = 2.*SMPRD*W2
      CON4 = -2.*PROD*W
      CON5 = SUM*W2
      CON6 = -SMPRD*W
      CON7 = SUM*W4
      CON8 = -SMPRD*W3
      CON9 = PROD*W2
      B21 = CONN*U(3)
      B22 = CONN*U(4)
      D1 = CON*U(2)
      D2 = CON*U(3)
      E11 = CON1*U(5) + CON2*U(4) + CON3*U(3) + CON4*U(2)
      E12 = CON1*U(6) + CON2*U(5) + CON3*U(4) + CON4*U(3)
      F21 = -W3*U(4) + CON5*U(3) + CON6*U(2) + PRDGN*U(1)
      F22 = -W3*U(5) + CON5*U(4) + CON6*U(3) + PRDGN*U(2)
      G1 = -W5*U(6) + CON7*U(5) + CON8*U(4) + CON9*U(3)
      G2 = -W5*U(7) + CON7*U(6) + CON8*U(5) + CON9*U(4)
      COF1 = B21*F22*(2.*F21*B22-F22*B21)-F21*(F21*B22*B22+D2*D2*B21)
      COF2 = E11*(2.*B22*(F22*B21-F21*B22)-D2*D2*B21)+2.*E12*B21*(F21*B2
     12-F22*B21)
      COF3 = B21*(-B21*(E12*E12+2.*F22*G2)-D2*D2*G1+2.*B22*(E11*E12+F21*
     1 G2+G1*F22))-B22*B22*(E11*E11+2.*F21*G1)
      COF4=2.*G2*B21*(E11*B22-E12*B21)+2.*B22*G1*(E12*B21-E11*B22)
      COF5= -(G2*B21-G1*B22)*(G2*B21-G1*B22)
      DO 50 I =1,5
   50 BCOFI(I) = 0.0
      BCOFR(1) = 1.0
      BCOFR(2) = COF2/COF1
      BCOFR(3) = COF3/COF1
      BCOFR(4) = COF4/COF1
      BCOFR(5) = COF5/COF1
      CALL ABETART
      IFLAG = 0
      CALL SORT
      IF (IFLAG-1)300,11,11
  300 BFIN(LJ) = BFINAL
      Q1 = BFIN(LJ)*(E11+BFIN(LJ)*F21)+G1
      ACOFR(1)=1.0
      ACOFR(2)=0.0
      ACOFR(3) = Q1/B21
      ALFASQ = ABSF(ACOFR(3))
      AFIN(LJ) = SQRTF(ALFASQ)                                          000790
   11 W = W+STP                                                         000800
   12 ZETA = ZETA + .0125                                               000810
      LBL(1) = 4HZ=.0                                                   000820
      LBL(2) = 4HZ=.1                                                   000830
      LBL(3) = 4HZ=.2                                                   000840
      LBL(4) = 4HZ=.3                                                   000850
      LBL(5) = 4HZ=.4                                                   000860
      LBL(6) = 4HZ=.5                                                   000870
      LBL(7) = 4HZ=.6                                                   000880
      LBL(8) = 4HZ=.7                                                   000890
      LBL(9) = 4HZ=.8                                                   000900
      LBL(10) = 4HZ=.9                                                  000910
      READ 7, ILGPLT                                                    000920
    7 FORMAT (I1)                                                       000930
      IF(ILGPLT-1)8,67,67                                               000940
    8 READ 3, (IT(I),I=1,12)                                            000950
    3 FORMAT (6A8)                                                      000960
      READ 6, (LBL(I),I=11,20)                                          000970
    6 FORMAT (10A4)                                                     000980
      READ 4, XLGZ,YLGZ                                                 000990
    4 FORMAT (2E10.0)                                                   001000
      XLGLM = 9.*XLGZ                                                   001010
      YLGLM = 15.*YLGZ                                                  001020
      MODE = 1                                                          001030
      IL = 0                                                            001040
      DO 62 K=1,80,8                                                    001050
      LL = 1                                                            001060
      KJ = (K-1)*80                                                     001070
      DO 61 J=1,80                                                      001080
      KJ = KJ+1                                                         001090
      IF(AFIN(KJ)-.0000001)61,6110,6110                                 001095
 6110 IF(AFIN(KJ)-1.0001)6113,61,61                                     001100
 6113 IF(AFIN(KJ) - XLGLM)6114,61,61                                    001110
C              CARDS 1120 - 1130 ARE MISSING
 6114 XAZ(LL) = AFIN(KJ)                                                001140
```

```
      IF(BFIN(KJ) - YLGLM)6112,61,61                                    001150
C           CARDS 1160 - 1170 ARE MISSING
 6112 YBZ(LL) = BFIN(KJ)                                                001180
      LL = LL + 1                                                       001190
   61 CONTINUE                                                          001200
      JJ = LL - 1                                                       001210
      IL = IL + 1                                                       001220
      IF(JJ-1)62,62,6116                                                001230
 6116 LAL = LBL(IL)                                                     001240
      CALL DRAW(JJ,XAZ,YBZ,MODE,0,LAL,IT,XLGZ,YLGZ,0,0,0,0,9,15,0,LAST)  001250
 6111 MODE = 2                                                          001260
   62 CONTINUE                                                          001270
      IF(MODE-1)65,65,6120                                              001280
 6120 DO 66 K=8,80,8                                                    001290
      LL = 1                                                            001300
      DO 63 J=1,80                                                      001310
      JK = (J-1)*80 + K                                                 001320
      IF(AFIN(JK)-.0000001)63,6127,6127                                 001325
 6127 IF(AFIN(JK)-1.0001)6123,63,63                                     001330
 6123 IF(AFIN(JK) - XLGLM)6124,63,63                                    001340
C          CARDS 1350 - 1360 ARE MISSING
 6124 XAW(LL) = AFIN(JK)                                                001370
      IF(BFIN(JK) - YLGLM)6122,63,63                                    001380
C             CARDS 1390 - 1400 ARE MISSING
 6122 YBW(LL) = BFIN(JK)                                                001410
      LL = LL + 1                                                       001420
   63 CONTINUE                                                          001430
      JJ = LL - 1                                                       001440
      IL = IL + 1                                                       001450
      IF(JJ-1)6121,6121,6126                                            001460
 6121 IF(K-80)66,6125,6125                                              001470
 6125 MODE = 3                                                          001480
      LAL = 4H                                                          001490
      JJ = 2                                                            001500
      XAW(1) = XLGLM                                                    001510
      XAW(2) = XLGLM                                                    001520
      YBW(1) = 0.0                                                      001530
      YBW(2) = YLGZ                                                     001540
      GO TO 2000                                                        001550
 6126 LAL = LBL(IL)                                                     001560
 2000 CALL DRAW(JJ,XAW,YBW,MODE,0,LAL,IT,XLGZ,YLGZ,0,0,0,0,9,15,0,LAST) 001570
      MODE = 2                                                          001580
      IF(K-72)66,64,64                                                  001590
   64 MODE = 3                                                          001600
   66 CONTINUE                                                          001610
      GO TO 67                                                          001620
   65 PRINT 130                                                         001630
  130 FORMAT (1X,33H NO LAG COMPENSATION IS POSSIBLE  ,///)             001640
   67 READ 20, ILDPLT                                                   001650
   20 FORMAT (I1)                                                       001660
      IF(ILDPLT-1)68,1000,1000                                          001670
   68 READ 5, (IT(I),I=1,12)                                            001680
    5 FORMAT (6A8)                                                      001690
      READ 21, XLDZ,YLDZ                                                001700
   21 FORMAT (2E10.0)                                                   001710
      READ 22, (LBL(I),I=11,20)                                         001713
   22 FORMAT (10A4)                                                     001716
      XLDLM = 9.*XLDZ                                                   001720
      YLDLM = 15.*YLDZ                                                  001730
      IL = 0                                                            001740
      MODE = 1                                                          001750
      DO 72 K = 1,80,8                                                  001760
      KJ = (K-1)*80                                                     001770
      KK = 1                                                            001780
      DO 71 J = 1,80                                                    001790
      KJ = KJ + 1                                                       001800
      IF(AFIN(KJ)-1.0001)71,7111,7111                                   001810
 7111 IF(AFIN(KJ) - XLDLM)7117,71,71                                    001820
C       CARDS 1830 - 1840 ARE MISSING
 7117 XAZ(KK) = AFIN(KJ)                                                001850
       IF(BFIN(KJ) - YLDLM)7118,71,71                                   001860
C      CARDS 1870 - 1880 ARE MISSING
```

```
 7118 YBZ(KK) = BFIN(KJ)                                                        001890
      KK = KK + 1                                                               001900
   71 CONTINUE                                                                  001910
      MM = KK-1                                                                 001920
      IL = IL + 1                                                               001930
      IF(MM-1)72,72,7119                                                        001940
 7119 LAL = LBL(IL)                                                             001950
      CALL DRAW(MM,XAZ,YBZ,MODE,0,LAL,IT,XLDZ,YLDZ,0,0,0,0,9,15,0,LAST)         001960
 7110 MODE = 2                                                                  001970
   72 CONTINUE                                                                  001980
      IF(MODE-1)70,70,78                                                        001990
   78 DO 76 K=8,80,8                                                            002000
      KK = 1                                                                    002010
      DO 73 J = 1,80                                                            002020
      JK = (J-1)*80 + K                                                         002030
      IF(AFIN(JK)-1.0001)73,7121,7121                                           002040
 7121 IF(AFIN(JK) - XLDLM)7127,73,73                                            002050
C      CARDS 2060 - 2070 ARE MISSING
 7127 XAW(KK) = AFIN(JK)                                                        002080
      IF(BFIN(JK) - YLDLM)7128,73,73                                            002090
C      CARDS 2100 - 2110 ARE MISSING
 7128 YBW(KK) = BFIN(JK)                                                        002120
      KK = KK + 1                                                               002130
   73 CONTINUE                                                                  002140
      MM = KK-1                                                                 002150
      IL = IL + 1                                                               002160
      IF(MM-1)7120,7120,7129                                                    002170
 7120 IF(K-80)76,7122,7122                                                      002180
 7122 MODE = 3                                                                  002190
      LAL = 4H                                                                  002200
      MM = 2                                                                    002210
      XAW(1) = XLDLM                                                            002220
      XAW(2) = XLDLM                                                            002230
      YBW(1) = 0.0                                                              002240
      YBW(2) = YLDZ                                                             002250
      GO TO 2001                                                                002260
 7129 LAL = LBL(IL)
 2001 CALL DRAW(MM,XAW,YBW,MODE,0,LAL,IT,XLDZ,YLDZ,0,0,0,0,9,15,0,LAST)
      MODE = 2
      IF(K-72)76,75,75
   75 MODE = 3
   76 CONTINUE
      GO TO 1000
   70 PRINT 131
  131 FORMAT (1X,34H NO LEAD COMPENSATION IS POSSIBLE  ,///)
 1000 CONTINUE
      ZLAB(1) = 0.0
      DO 81 I=2,10
   81 ZLAB(I) = ZLAB(I-1) + .1
      WLAB(8) = 8.*STP
      DO 82 N=16,80,8
   82 WLAB(N) = WLAB(N-8) + WLAB(8)
      PRINT 100
  100 FORMAT (1H1)
      PRINT 101
  101 FORMAT(2X,20H THE ALFA VALUES ARE  ,//)
      PRINT 102, (ZLAB(I) I=1,10)
  102 FORMAT (1X,6H ZETA   ,10F11.6)
      PRINT 103, (WLAB(J),(AFIN(J,I),I=1,73,8)J=8,80,8)
  103 FORMAT (/,1X,F6.2,10E11.5)
      PRINT 111
  111 FORMAT (/////2X,20H THE BETA VALUES ARE      ,//)
      PRINT 112, (ZLAB(I) I=1,10)
  112 FORMAT (1X,6H ZETA   ,10F11.6)
      PRINT 113, (WLAB(J),(BFIN(J,I),I=1,73,8)J=8,80,8)
  113 FORMAT (/,1X,F6.2,10E11.5)
   24 PRINT 114
  114 FORMAT (1H1)
      READ 218, IBWCMP
  218 FORMAT (I1)
      IF(IBWCMP-1)1002,219,219
  219 READ 217, BWX,BWY
```

```
217 FORMAT (2E10.0)                                                   002630
    READ 223, WEND                                                    002633
223 FORMAT (F10.0)                                                    002636
    YBWLM = 15.*BWY                                                   002640
    XLM = 9.*BWX                                                      002650
    AM2=.5                                                            002660
    STEP = WEND/20.                                                   002670
    W = STEP                                                          002680
    ALFASP = XLM/20.                                                  002690
    XAW(1) = ALFASP                                                   002700
    DO 200 K=2,20                                                     002710
200 XAW(K) = XAW(K-1) + XAW(1)                                        002720
    DO 203 N=1,20                                                     002730
    ALFA = ALFASP                                                     002740
    DO 210 M=1,20                                                     002750
    XAZ(M) = 0.0                                                      002760
210 YBZ(M) = 0.0                                                      002770
    DO 207 I=1,20                                                     002780
    YBW(N) = W                                                        002790
    W2=W*W                                                            002800
    W3=W*W2                                                           002810
    W4=W2*W2                                                          002820
    W5=W2*W3                                                          002830
    AGR = -W2*G                                                       002840
    AKR = G                                                           002850
    ADR=2.*W4-2.*W2*SMPRD                                             002860
    AII = 2.*G*W                                                      002870
    AER=-W2*SUM + PROD                                                002880
    AFR=W4*SUM - W2*PROD                                              002890
    ADI=-2.*W3*SUM + 2.*W*PROD                                        002900
    AEI = -W3 + W*SMPRD                                               002910
    AFI = W5 - W3*SMPRD                                               002920
    P1 = ADR*ADR+ADI*ADI                                              002930
    Q1 = 2.*ADR*AER + 2.*AEI*ADI                                      002940
    R1 = 2.*ADR*AFR + AER*AER + AEI*AEI + 2.*ADI*AFI                  002950
    V1 = 2.*AER*AFR + 2.*AEI*AFI                                      002960
```

```
      W1 = AFR*AFR + AFI*AFI
      A2 = ALFA*ALFA
      A4 = A2*A2
      R2 = A2*AII*AII
      WW2 = A4*AGR*AGR + 2.*A2*AKR*AGR + AKR*AKR
      DO 51 M = 1,5
   51 BCOFI(M) = 0.0
      BCOFR(1) = 1.0
      BCOFR(2) = Q1/P1
      BCOFR(3) = (AM2*R1-R2)/(AM2*P1)
      BCOFR(4) = V1/P1
      BCOFR(5) = (AM2*W1-WW2)/(AM2*P1)
      CALL ABETART
      IFLAG = 0
      CALL SORT
      IF(IFLAG-1)900,206,206
  206 BFIN(N,I) = 0.0
      GO TO 207
  900 BFIN(N,I) = BFINAL
  207 ALFA = ALFA + ALFASP
  203 W = W + STEP
      READ 216, IBWPLT
  216 FORMAT (I1)
      IF(IBWPLT-1)214,1001,1001
  214 MODE = 1
      READ 202,(IT(K),K=1,12)
  202 FORMAT (6A8)
      READ 201, (LBL(N),N=2,20,2)
  201 FORMAT (10A4)
      DO 211 N=2,20,2
      KK = 1
      IF(N-20)204,205,205
  205 MODE = 3
  204 CONTINUE
      DO 212 I=1,20
      IF(BFIN(N,I) - .000001)212,212,209
  209 IF(BFIN(N,I) - YBWLM)905,906,906                                   003330
  905 YBZ(KK) = BFIN(N,I)                                                003340
      XAZ(KK) = XAW(I)                                                   003350
C      CARD 3360 IS MISSSING
      KK = KK + 1                                                        003370
  212 CONTINUE                                                           003380
  906 JJ = KK - 1                                                        003390
      IF(JJ-1)221,221,220                                                003400
  221 IF(N-20)211,225,225                                                003410
  225 IF(MODE-1)224,224,215                                              003415
  215 MODE = 3                                                           003420
      LAL = 4H                                                           003430
      XAZ(1) = 0.0                                                       003440
      XAZ(2) = 0.0                                                       003450
      YBZ(1) = 0.0                                                       003460
      YBZ(2) = BWY                                                       003470
      JJ = 2                                                             003480
      GO TO 2002                                                         003490
  220 LAL = LBL(N)                                                       003500
 2002 CALL DRAW (JJ,XAZ,YBZ,MODE,0,LAL,IT,BWX,BWY,0,0,0,0,9,15,0,LAST)   003510
  222 IF(N-20)208,211,211                                                003520
  208 MODE = 2                                                           003530
  211 CONTINUE                                                           003540
      GO TO 1001                                                         003541
  224 PRINT 226                                                          003542
  226 FORMAT (1H1,1X,76HTHERE ARE NO POSSIBLE BANDWIDTH CURVES FOR THE F 003543
     1INAL VALUE OF OMEGA STATED     )                                   003544
      GO TO 1002                                                         003545
 1001 PRINT 120                                                          003550
  120 FORMAT (1H1,20X,38H VALUES OF BETA FOR CONSTANT BANDWIDTH       )  003560
      PRINT 122, (YBW(K),K=2,20,2)                                       003570
  122 FORMAT (/,9X,10F11.6)                                              003580
      PRINT 121, (XAW(K),(BFIN(J,K),J=2,20,2),K=1,20)                    003590
  121 FORMAT (/,1X,F6.2,2X,10E11.6)                                      003600
 1002 CONTINUE                                                           003610
      GO TO 9999                                                         003615
      END                                                                    003620
                                                                             00363
                                                                             00364
      SUBROUTINE ABETART                                                     003650
      DIMENSION A(5),YIMAG(5),U(4),V(4),H(50),B(50),C(50),D(50),E(50)       003660
     1 ,CONV(50)                                                             003670
      DIMENSION AFIN(80,80),BFIN(80,80)                                      003680
      COMMON A,YIMAG,U,V,DUMMY1,DUMMY2,AFIN,BFIN                             003690
      N = 4                                                                  003700
      F=10.0                                                               00003710
      L=25                                                                 00003720
      IER=0                                                                00003730
      IF(N) 54,54,52                                                       00003740
   54 IER=1                                                                00003750
   52 NP3=N+3                                                              00003760
  100 B(2)=0.0                                                             00003770
      B(1)=0.0                                                             00003780
      C(2)=0.0                                                             00003790
      C(1)=0.0                                                             00003800
      D(2)=0.0                                                             00003810
      E(2)=0.0                                                             00003820
      H(2)=0.0                                                             00003830
      DO 101  J=3,NP3                                                      00003840
  101 H(J)=A(J-2)                                                          00003850
      T=1.0                                                                00003860
      SK=10.0**F                                                           00003870
  150 IF(H(NP3)) 200,151,200                                               00003880
  151 U(NP3)=0.0                                                           00003890
      V(NP3)=0.0                                                           00003900
      CONV(NP3)=SK                                                         00003910
      NP3=NP3-1                                                            00003920
      IF(NP3)152,152,150                                                   00003930
  152 IER=1                                                                00003940
  200 IF(NP3-3)205,51,201                                                  00003950
  205 IER=1                                                                00003960
  201 PS=0.0                                                               00003970
```

```
      QS=0.0
      PT=0.0
      QT=0.0
      S=0.0
      REV=1.0
      SK=10.0**F
      IF(NP3-4)206,202,203
  206 IER=1
  202 R=-H(4)/H(3)
      GO TO 500
  203 DO 207 J=3,NP3
      IF(H(J))204,207,204
  204 S=S+LOGF(ABSF(H(J)))
  207 CONTINUE
      FPN1=N+1
      S=EXPF(S/FPN1)
      DO 208 J=3,NP3
  208 H(J)=H(J)/S
  210 IF(ABSF(H(4)/H(3))-ABSF(H(NP3-1)/H(NP3)))250,252,252
  250 T=-T
      M=(NP3-4)/2 + 3
      DO 251 J=3,M
      S=H(J)
      JJ=NP3-J+3
      H(J)=H(JJ)
  251 H(JJ)=S
  252 IF(QS) 253,254,253
  253 P=PS
      Q=QS
      GO TO 300
  254 HH2=H(NP3-2)
      IF(HH2) 256,255,256
  255 Q=1.0
      P=-2.0
      GO TO 257
  256 Q=H(NP3)/HH2
```

```
      P=(H(NP3-1)-Q*H(NP3-3))/HH2                                        00004340
  257 IF(NP3-5)258,550,258                                               00004350
  258 R=0.0                                                              00004360
  300 DO 490  I=1,L                                                      00004370
  350 DO 351  J=3,NP3                                                    00004380
      B(J)=H(J)-P*B(J-1)-Q*B(J-2)                                        00004390
  351 C(J)=B(J)-P*C(J-1)-Q*C(J-2)                                        00004400
      IF(H(NP3-1))352,400,352                                            00004410
  352 IF(B(NP3-1))353,400,353                                            00004420
  353 AVHB1=ABSF(H(NP3-1)/B(NP3-1))                                      00004430
  356 IF(AVHB1-SK)450,354,354                                            00004440
  354 B(NP3)=H(NP3)-Q*B(NP3-2)                                           00004450
  400 IF(B(NP3))401,550,401                                              00004460
  401 AVHB2=ABSF(H(NP3)/B(NP3))                                          00004470
  403 IF(SK-AVHB2)550,450,450                                            00004480
  450 DO 451  J=3,NP3                                                    00004490
      D(J)=H(J)+R*D(J-1)                                                 00004500
  451 E(J)=D(J)+R*E(J-1)                                                 00004510
      IF(D(NP3))452,500,452                                              00004520
  452 AVHD3=ABSF(H(NP3)/D(NP3))                                          00004530
  460 IF(SK-AVHD3)500,453,453                                            00004540
  453 CC2=C(NP3-2)                                                       00004550
      CC3=C(NP3-3)                                                       00004560
      C(NP3-1)=-P*CC2-Q*CC3                                              00004570
      CC1=C(NP3-1)                                                       00004580
      S=CC2*CC2-CC1*CC3                                                  00004590
      IF(S)455,454,455                                                   00004600
  454 P=P-2.0                                                            00004610
      Q=Q*(Q+1.0)                                                        00004620
      GO TO 456                                                          00004630
  455 P=P+(B(NP3-1)*CC2-B(NP3)*CC3)/S                                    00004640
      Q=Q+(-B(NP3-1)*CC1+B(NP3)*CC2)/S                                   00004650
  456 IF(E(NP3-1))458,457,458                                            00004660
  457 R=R-1.0                                                            00004670
      GO TO 490                                                          00004680
  458 R=R-D(NP3)/E(NP3-1)                                                00004690
```

```
490 CONTINUE                                                    00004700
    PS=PT                                                       00004710
    QS=QT                                                       00004720
    PT=P                                                        00004730
    QT=Q                                                        00004740
    IF(REV)491,492,492                                          00004750
491 SK=SK/10.0                                                  00004760
492 REV=-REV                                                    00004770
    GO TO 250                                                   00004780
500 IF(T)501,502,502                                            00004790
501 R=1.0/R                                                     00004800
502 NP=NP3-3                                                    00004810
    U(NP)=R                                                     00004820
    V(NP)=0.0                                                   00004830
    CONV(NP)=SK                                                 00004840
    NP3=NP3-1                                                   00004850
    DO 503 J=3,NP3                                              00004860
503 H(J)=D(J)                                                   00004870
    IF(NP3-3)300,51,300                                         00004880
550 IF(T)551,552,552                                            00004890
551 P=P/Q                                                       00004900
    Q=1.0/Q                                                     00004910
552 PP2=P/2.0                                                   00004920
    QMPSQ=Q-PP2*PP2                                             00004930
560 IF(QMPSQ)554,554,553                                        00004940
553 NP=NP3-3                                                    00004950
    U(NP)=-PP2                                                  00004960
    U(NP-1)=-PP2                                                00004970
    S=SQRTF(QMPSQ)                                              00004980
    V(NP)=S                                                     00004990
    V(NP-1)=-S                                                  00005000
    GO TO 561                                                   00005010
554 S=SQRTF(-QMPSQ)                                             00005020
    NP=NP3-3                                                    00005030
    IF(P)555,556,556                                            00005040
555 U(NP)=-PP2+S                                                00005050
```

```
    GO TO 557
556 U(NP)=-PP2-S
557 U(NP-1)=Q/U(NP)
    V(NP)=0.0
    V(NP-1)=0.0
561 CONV(NP)=SK
    CONV(NP-1)=SK
    NP3=NP3-2
    DO 558 J=3,NP3
558 H(J)=B(J)
    GO TO 200
 51 RETURN
    END


    SUBROUTINE SORT
    DIMENSION RR(4),RI(4),REAL(5),YIMAG(5)
    DIMENSION AFIN(80,80),BFIN(80,80)
    COMMON REAL,YIMAG,RR,RI,BF,IFLAG,AFIN,BFIN
    B = 0.0
    DO 800 I=1,4
    IF(ABSF(RI(I))-1.E-7)801,800,800
801 B = MAX1F(B,RR(I))
800 CONTINUE
    IF(B) 802,802,803
802 PRINT 804
804 FORMAT (34H THERE ARE NO POSITIVE REAL ROOTS   )
    IFLAG = 1
     RETURN
803 BF = B
     RETURN
    END
    END
```

APPENDIX IV

PROGRAM PLOT

```
..JOB0473F, PAINE, J.A.,JR. - THESIS
      PROGRAM PLOT
C    THE ORDER OF THE DATA CARDS IS
C      CARD 1 - UNCOMPENSATED POLE LOCATIONS  (3E10.0)
C      CARD 2 - VALUES OF ALFA AND BETA  (2E20.0)
C      CARD 3 FIRST LINE OF GRAPH TITLE (COLUMNS 1-48)
C      CARD 4 SECOND LINE OF GRAPH TITLE (COLUMNE 1-48)
C      CARD 5 INITIAL VALUE OF THE VARIABLE (E10.5) MAY NOT BE ZERO
C      CARD 6 NUMBER OF DECADES TO BE SPANNED (1 THRU 10)  (I3)
C      CARD 7 GRAPH SCALE (E10.5) (ONE SIGNIFICAT FIGURE)
C   THE PROGRAM WILL DETERMINE THE VALUES OF THE COEFFICIENTS AND PUT
C         THEM IN TO THE RTLOCUS PROGRAM
      DIMENSION A(6),B(6)
    3 READ 1,P,Q,R
    1 FORMAT (3E10.0)
      READ 2, ALFA,BETA
    2 FORMAT (2E20.0)
      SUM = P+Q+R
      SMPRD = P*Q + P*R + Q*R
      PROD = P*Q*R
      A(1) = 1.0
      A(2) = 2.*BETA + SUM
      A(3) = BETA*BETA + 2.*BETA*SUM + SMPRD
      A(4) = BETA*BETA*SUM + 2.*BETA*SMPRD + PROD
      A(5) = BETA*BETA*SMPRD + 2.*BETA*PROD
      A(6) = BETA*BETA*PROD
      B(1) = 0.0
      B(2) = 0.0
      B(3) = 0.0
      B(4) = ALFA*ALFA
      B(5) = 2.*BETA*ALFA
      B(6) = BETA*BETA
      CALL PLTRTS (A,B)
      GO TO 3
      END
      SUBROUTINE PLTRTS (A,B)
      DIMENSION R(129),X(129),IT(10),ROOTR(128),ROOTI(128),ITITLE(12),
     1A(129),B(129),ROOTJ(128),ROOTM(128),AP(129),AZ(129)
      COMMON R,VAR,NO,ROOTR,ROOTI,AP,X
      MOD = 1
      LAB=4H
   68 FORMAT(8E15.5)
   70 FORMAT(////,14HIMAGINARY PART,//)
   69 FORMAT(////,9HREAL PART,//)
  200 FORMAT(6A8)
  203 FORMAT(I3)
  204 FORMAT(E10.5)
      READ 200,(ITITLE(I),I=1,6)
      READ 200,(ITITLE(I),I=7,12)
   24 FORMAT(1H1,////,17HTHE INPUT DATA IS,////)
      PRINT 24
  324 FORMAT(///,11HGRAPH TITLE,///)
      PRINT 324
      PRINT 200,(ITITLE(I),I=1,6)
      PRINT 200,(ITITLE(I),I=7,12)
   28 FORMAT(///,36HORDER OF THE CHARACTERISTIC EQUATION,///)
      PRINT 28
      NO = 5
      PRINT 203,NO
      N=NO+1
  205 FORMAT(8E10.5)
  207 FORMAT (8E12.5)
   22 FORMAT(///,53HCONSTANT PART OF THE COEFFICIENTS IN DESCENDING ORDE
     1R,///)
      PRINT 22
      PRINT 207,(A(K),K=1,N)
   23 FORMAT(///,53HVARIABLE PART OF THE COEFFICIENTS IN DESCENDING ORDE
     1R,///)
      PRINT 23
      PRINT 207,(B(K),K=1,N)
   25 FORMAT(///,29HINITIAL VALUE OF THE VARIABLE,///)
      PRINT 25
      READ 204,VAR
      PRINT 204,VAR
   26 FORMAT(///,31HNUMBER OF DECADES TO BE SPANNED,///)
      PRINT 26
      READ 203,ND
      PRINT 203,ND
   27 FORMAT(///,5HSCALE,///)
      PRINT 27
      READ 204,XSCALE
      PRINT 204,XSCALE
      YSCALE=XSCALE
  201 FORMAT(21H1THE SYSTEM POLES ARE,////)
      PRINT 201
      M=N
      DO 67 K=1,N
      AP(K)=A(M)
   67 M=M-1
      DO 432 K=1,30
      ROOTI(K)=0.
  432 ROOTR(K)=0.
      NX=NO
      DO 15 K=1,31
   15 X(K)=0.0
      CALL POLYRT (AP,X,NX,ROOTR,ROOTI,1.E-5)
      PRINT 69
      PRINT 68,(ROOTR(K),K=1,NO)
      PRINT 70
      PRINT 68,(ROOTI(K),K=1,NO)
      CALL DRAW(NO,ROOTR,ROOTI,MOD,1,LAB,ITITLE,XSCALE,YSCALE,
     11,6,2,2,7,8,1,LAST)
      MOD=2
  202 FORMAT(//,21H THE SYSTEM ZEROS ARE,///)
      K=1
    3 IF(B(K)) 1,2,1
    2 K=K+1
      GO TO 3
    1 NORD=N-K
      IF(NORD-1) 6,4,5
    4 ZERO=-B(K+1)/B(K)
    7 FORMAT(//,16HTHE SYSTEM ZERO=,E15.5,///)
      PRINT 7,ZERO
      ROOTM(1)=ZERO
      ROOTM(2)=16.*XSCALE
      ROOTJ(1)=0.
      ROOTJ(2)=0.
      NORD=2
      GO TO 11
    6 PRINT 9
    9 FORMAT(//,25HALL ZEROS ARE AT INFINITY)
      GO TO 8
    5 NN=NORD+1
      DO 10 L=1,NN
      R(L)=B(K)
   10 K=K+1
      PRINT 202
      M=NN
      DO 46 K=1,NN
      AZ(K)=R(M)
   46 M=M-1
      DO 433 K=1,30
      ROOTM(K)=0.
  433 ROOTJ(K)=0.
      NORX=NORD
      DO 16 K=1,31
   16 X(K)=0.0
      CALL POLYRT(AZ,X,NORX,ROOTM,ROOTJ,1.E-5)
      PRINT 69
      PRINT 68,(ROOTM(K),K=1,NORD)
      PRINT 70
      PRINT 68,(ROOTJ(K),K=1,NORD)
   11 CALL DRAW(NORD,ROOTM,ROOTJ,MOD,3,LAB,ITITLE,XSCALE,YSCALE,
     11,6,2,2,7,8,1,LAST)
    8 CONTINUE
      GO TO(31,32,33,34,35,36,37,38,39,40),ND
   31 G=1.0076
      GO TO 41
   32 G=1.016
      GO TO 41
   33 G=1.0245
      GO TO 41
   34 G=1.0312
      GO TO 41
   35 G=1.0394
      GO TO 41
   36 G=1.0483
      GO TO 41
   37 G=1.0568
      GO TO 41
   38 G=1.0633
      GO TO 41
   39 G=1.071
      GO TO 41
   40 G=1.078
   41 PRINT 30
   30 FORMAT(1H1,////,61HROOTS FOR THE SPECIFIED VALUES OF THE VARIABLE
     1ARE AS FOLLOWS,////)
      PRINT 42
   42 FORMAT(10X,4H VAR,4X,9HREAL PART,3X,9HIMAG PART,
     113X,9HREAL PART,3X,9HIMAG PART,13X,9HREAL PART,3X,9HIMAG PART,///)
      DO 101 J=1,30
      DO 100 K=1,10
   60 FORMAT(1PE12.3)
      PRINT 60,VAR
      DO 300 L=1,N
  300 R(L)=A(L)+B(L)*VAR
   50 FORMAT(15X,3(1P2E12.3,10X))
      CALL ROOTX
      DO 71 JJ=1,NO
      IF(ABSF(ROOTI(JJ))-5.E-04) 61,61,62                               001780
   61 ROOTJ(JJ)=0.                                                      001790
      GO TO 71                                                          001800
   62 ROOTJ(JJ)=ROOTI(JJ)                                               001810
   71 CONTINUE                                                          001820
      PRINT 50,(ROOTR(I),ROOTJ(I),I=1,NO)                               001830
  100 VAR=VAR*G                                                         001840
      IF(J-30) 101,98,98                                                001850
   98 MOD=3                                                             001860
  101 CALL DRAW(NO,ROOTR,ROOTJ,MOD,2,LAB,ITITLE,XSCALE,YSCALE,          001870
     1 1,6,2,2,7,8,1,LAST)                                              001880
      END                                                               001890
      SUBROUTINE ROOTX                                                  001900
      DIMENSION C(31),D(29),R(129),ROOTR(128),ROOTI(128),EE(31),FE(31)  001910
     1,AP(129),X(129)                                                   001920
      COMMON R,VAR,NO,ROOTR,ROOTI,AP,X                                  001930
      M=1                                                               001940
      DO 1492 MA=1,31                                                   001950
      EE(MA)=0.                                                         001960
 1492 FE(MA)=0.                                                         001970
   20 BETAN=ROOTI(M)+EE(M)                                              001980
      ALFAN=ROOTR(M)+FE(M)                                              001990
      DO 7 I=1,100                                                      002000
      S=2.*ALFAN                                                        002010
      T=-(ALFAN**2+BETAN**2)                                            002020
      C(1)=R(1)                                                         002030
      C(2)=R(2)+S*R(1)                                                  002040
      NC=NO+1                                                           002050
      DO 2 L=3,NC                                                       002060
    2 C(L) = R(L)+S*C(L-1) + T*C(L-2)                                   002070
      AN= C(NO+1)-ALFAN*C(NO)                                           002080
      BN= BETAN*C(NO)                                                   002090
      IF (NO-3) 21, 17, 18                                              002100
   17 CN = 3.*R(1)*(ALFAN**2-BETAN**2) + 2.*R(2)*ALFAN + R(3)           002110
      DN = 6.*R(1)*ALFAN*BETAN + 2.*R(2)*BETAN                          002120
      GO TO 19                                                          002130
   21 CN = 2.*R(1)*ALFAN + R(2)
      DN  = 2.*R(1)*BETAN
      GO TO 19
   18 D(1) = C(1)
      D(2)=C(2)+S*D(1)
      NU=NO-1
      DO 3 N=3,NU
    3 D(N)=C(N)+S*D(N-1)+T*D(N-2)
      CN= C(NO)-2.*D(NO-2)*BETAN**2
      DN= 2.*BETAN*(D(NO-1)-ALFAN*D(NO-2))
   19 ALFA=ALFAN-(AN*CN+BN*DN)/(CN**2+DN**2)
      BETA =BETAN + (AN*DN-BN*CN)/(CN**2+DN**2)
      AZETA=.01+ABSF(BETAN)
      ZETA=SQRTF(.01-T)
      IF(ABSF((ALFA-ALFAN)/ZETA )-.1)55,55,52
   55    IF(ABSF((BETA-BETAN)/AZETA)-1.)56,56,52
   56 EE(M) =(ALFA-ALFAN)*10. + BETA - ROOTI(M)
      FE(M)=(BETA - BETAN)*10. +ALFA - ROOTR(M)
      IF(ABSF((ALFA-ALFAN)/(ALFAN+1.))-5.E-4)4,4,5
    4 IF(ABSF((BETA-BETAN)/(BETAN+1.))-5.E-4)6,6,5
    5 ALFAN=ALFA
    7 BETAN=BETA
   52 CONTINUE
      MR=NC
      DO 6777 K=1,NC
      AP(K)=R(MR)
 6777 MR=MR-1
      DO 434 K=1,30
      ROOTI(K)=0.
  434 ROOTR(K)=0.
      NX=NO
      DO 83 K=1,31
   83 X(K)=0.0
      CALL POLYRT (AP,X,NX,ROOTR,ROOTI,1.E-5)
      GO TO 16
    6 ROOTR(M)=ALFA
 1 ROOTI(M)=BETA
12 IF(M-NO) 15,16,16
15 M=M+1
   GO TO 20
16 RETURN
   END
   END
```

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Parameter plane techniques are applied to compensation of third order control systems by extending the principle to include the quadratic form of the parameters.

Equations for constant bandwidth curves on the parameter plane are developed.

Recursive equations for the extension of the method to higher ordered systems are presented.

A computer program for the computation and plotting of constant zeta, omega and bandwidth loci is written.

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